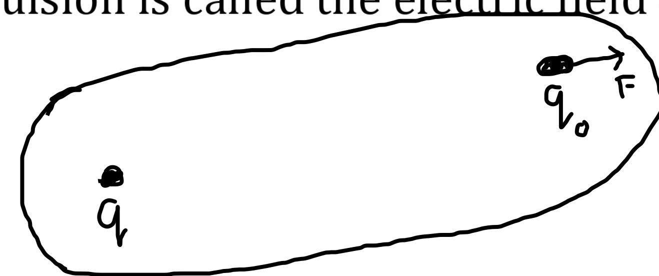
Electric Field:

The space surrounding an electric charge ‘q’ in which another charge q_0 experiences a force of attraction or repulsion is called the electric field of the charge q.



The charge q is “source charge” and the charge q_0 is called “test charge”. The source charge may be point charge, a group of point charge or continuous charge.

Intensity of Electric Field ($\vec{E}$)

Electric field intensity at any point is can be defined as force experienced per unit positive test charge placed at that point without disturbing the source charge.

If $\vec{F}$ is force experienced by test charge q_0, placed at any point in electric field then the Electric field intensity $\vec{E}$ at that point is given by,

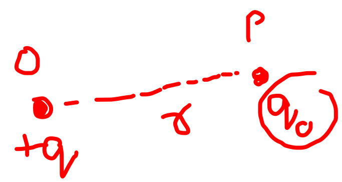
$$\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$$

SI unit is N/C

The direction of $\vec{E}$ is in the direction of the force $\vec{F}$.

<u>Electric field intensity due to a point charge</u>

Let us consider an isolated point charge of +q at O in a vacuum. Let q_0 be the test charge placed at a point P at which we have to determine the intensity of the electric field.



now, the electric force acting on q_0 is $\left|\vec{F}\right| = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r^2}$

hence, the intensity of the electric field at the point P is given by,

$$\left|\overrightarrow{E}\right| = \frac{\left|\vec{F}\right|}{q_0} = \frac{\frac{1}{4\pi\epsilon_0}\frac{q q_0}{r^2}}{q_0}$$

$$Or, \boxed{\left|\overrightarrow{E}\right| = \frac{1}{4\pi\epsilon_0}\,\frac{q}{r^2}}$$

If charges are kept in a material with dielectric constant K, then

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0 K}\frac{q}{r^2}$$

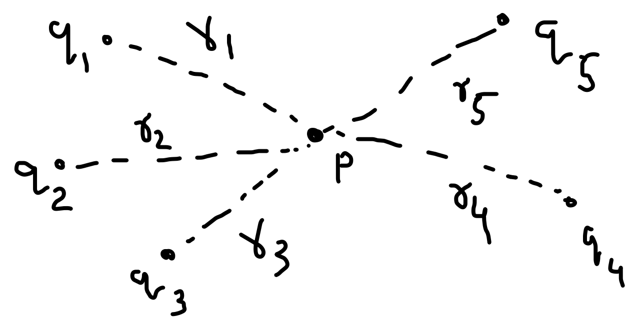
K=1
K>1

$$In\ vectors, \qquad \vec{E} = \frac{1}{4\pi\epsilon_0 K}\frac{q}{r^2}\hat{r}$$

<u>Electric Field intensity due to a groups of point charges</u>

Let us consider a system of 'n' point charges $q_1, q_2, q_3 \dots, q_n$ be distributed in space in a discrete manner around point P. Let electric field intensity due to $q_1, q_2, q_3 \dots, q_n$ are $E_1, E_2, E_3 \dots E_n$ at point P respectively.

Then according to the superposition principle, the total electric field at P is given by,



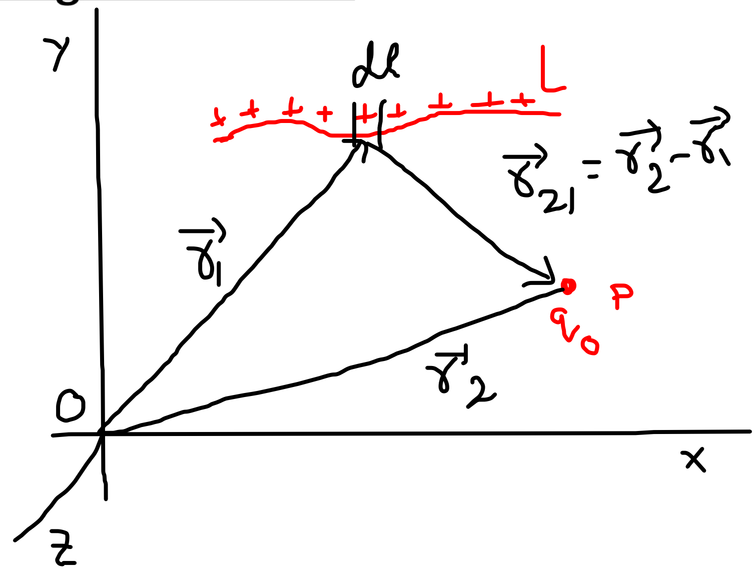
$$\vec{E}_{net} = \overrightarrow{E_1} + \overrightarrow{E_2} + \cdots + \overrightarrow{E_n}$$

$$\boxed{\vec{E}_{net} = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i}$$

Intensity of Electric field due to continuous charge distribution

i) Linear charge distribution

Let us consider a line charge (L) of linear charge density λ.
Let dl be an infinitesimally small element of this charge.
so, the charge on the element dl is,



$$dq = \lambda dl \qquad \left(\because \lambda = \frac{dq}{dl} \right)$$

Let $\vec{r}_1$ is position vector of dl with respect to origin

$\vec{r}_2$ is position vector of q_0 at P with respect to origin

now, Force on q_0 due to dq on length element dl is given by:

$$d\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_0 dq}{r_{21}^2} \widehat{r_{21}}$$

Hence, force on q_0 due to whole line charge,

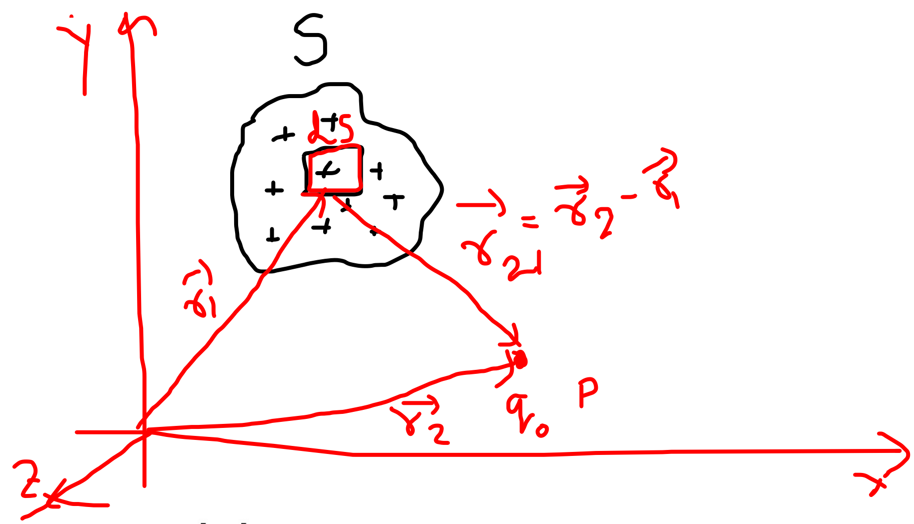
$$\vec{F} = \frac{q_0}{4\pi\epsilon_0}\int_L \frac{\lambda dl}{r_{21}^2}\widehat{r_{21}}$$

then, electric field due to line charge is given by,

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_L \frac{\lambda dl}{r_{21}^2}\widehat{r_{21}}$$

$\left(\because E = \frac{F}{q_0}\right)$

ii) Surface charge distribution



Let us consider a surface charge (S) of surface charge density σ.
Let ds be an infinitesimally small surface element of this charge.
so, the charge on the element ds is,

$$dq = \sigma ds$$

Let $\vec{r}_1$ is position vector of ds with respect to origin

$\vec{r}_2$ is position vector of q_0 at P with respect to origin now, Force on q_0 due to dq on element ds is given by:

$$d\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_0 dq}{r_{21}^2}\widehat{r_{21}}$$

Hence, force on q_0 due to whole surface charge,

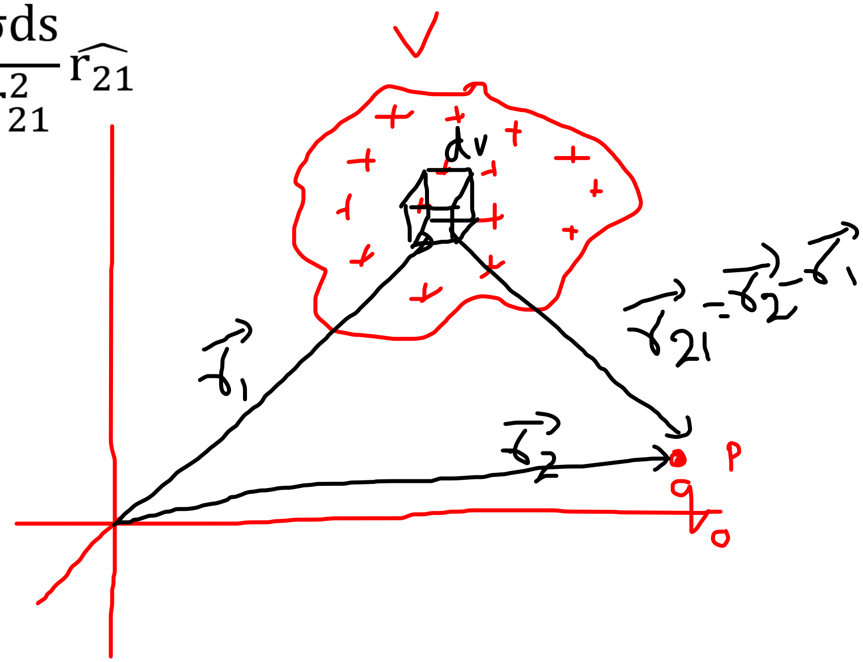
$$\vec{F} = \frac{q_0}{4\pi\epsilon_0}\int_S \frac{\sigma\, ds}{r_{21}^2}\widehat{r_{21}}$$

then, electric field due to surface charge is given by,

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_S \frac{\sigma ds}{r_{21}^2}\widehat{r_{21}}$$

i) Volume charge distribution

Let us consider a volume charge (V) of volume charge density ρ.
Let dv be an infinitesimally small volume element of this charge.



so, the charge on the element dv is,

$$dq = \rho dv$$

Let $\vec{r}_1$ is position vector of dv with respect to origin

$\vec{r}_2$ is position vector of q_0 at P with respect to origin

now, Force on q_0 due to dq on element dv is given by:

$$d\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_0 dq}{r_{21}^2}\widehat{r_{21}}$$

Hence, force on q_0 due to whole volume charge,

$$\vec{F} = \frac{q_0}{4\pi\epsilon_0}\int_V \frac{\rho\, dv}{r_{21}^2}\widehat{r_{21}}$$

then, electric field due to volume charge is given by,

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho dv}{r_{21}^2}\widehat{r_{21}}$$

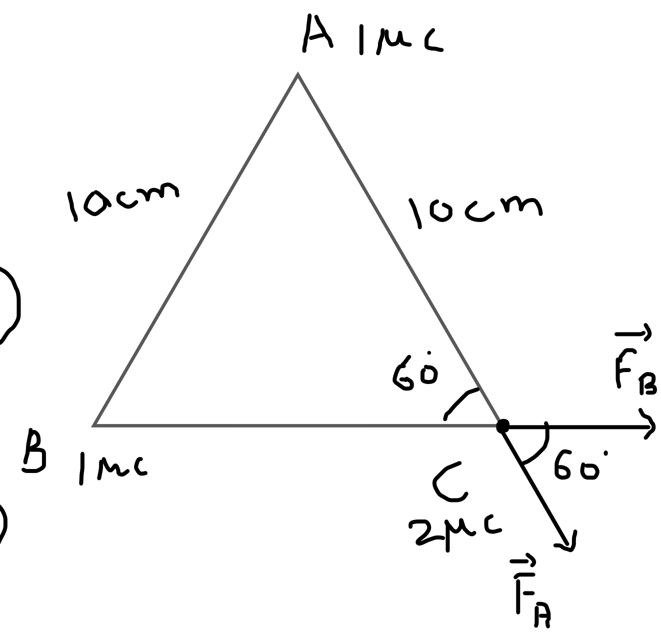
Que 8: Three charges 1μC, 1μC and 2μC are kept at vertices A, B and C of an equilateral triangle ABC of 10 cm side respectively. What will the resultant force on the charge at C?

Sol:

$$\vec{F}_A = \frac{1}{4\pi\epsilon_0} \frac{q_A q_C}{(10\times10^{-2})^2} \quad \text{along } AC$$

$$\vec{F}_A = 9\times10^9 \frac{1\times10^{-6}\times2\times10^{-6}}{100\times10^{-4}} = x$$

$$\vec{F}_B = 9\times10^9 \frac{1\times10^{-6}\times2\times10^{-6}}{100\times10^{-4}} = x$$



$$F = \sqrt{F_A^2 + F_B^2 + 2F_A F_B \cos\theta}$$

$\theta = 60°$

F = ?

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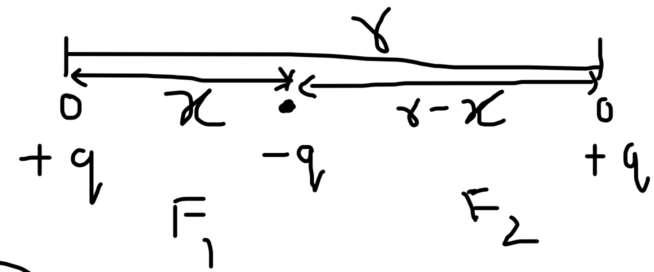
Que 9: Two charges each +q C are placed along line. A third charge -q is placed between them. At what position will the system be in equilibrium?

$$F_1 = 9 \times 10^9 \frac{qq}{x^2} \quad \text{—(1)}$$



$$F_2 = 9 \times 10^9 \frac{qq}{(r-x)^2} \quad \text{—(11)}$$

For equilibrium

$$F_1 = F_2$$

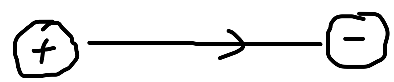
$$x = ?$$

Electric line of forces (OR Field lines):

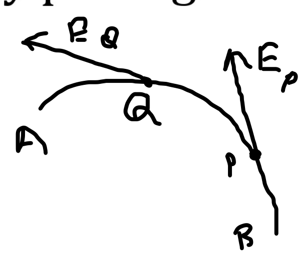
Electric field lines are defined as straight or curved path such that tangent to it at any point gives the direction of electric field intensity at that point.

Properties of electric field lines

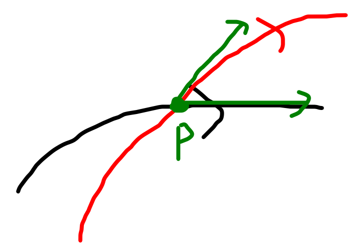
i) Electric line of forces are discontinuous curves. i.e. they start from positive charge and ends at negative charge.



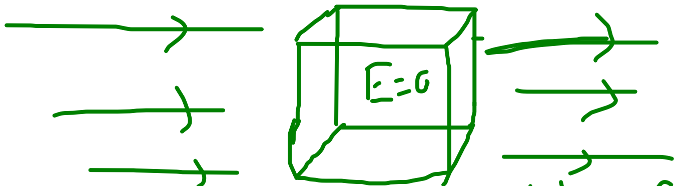
ii) Tangents to the lines of forces at any point gives the direction of electric intensity.

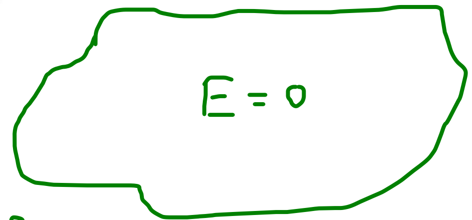


iii) No two electric line can intersect each other.



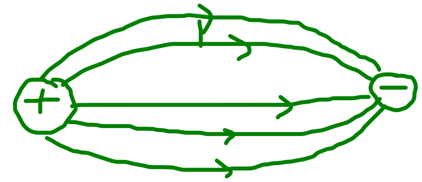
iv) The electric lines are always zero inside a conductor because they do not pass through a conductor.



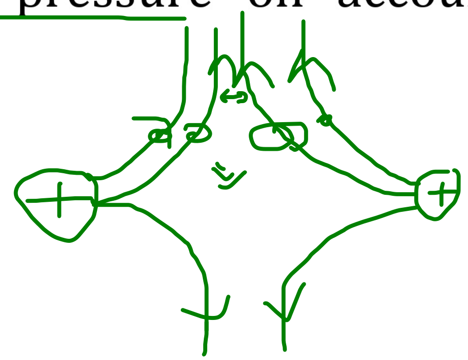


Electrostatic Shielding

v) The electric line contract longitudinally on account of attraction between unlike charges.

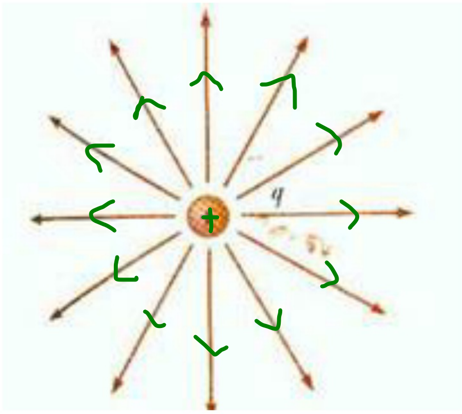


vi) The electric line exerts a lateral pressure on account of repulsion between like charges.

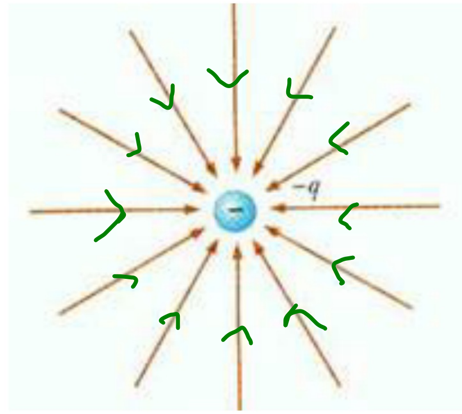


Examples:

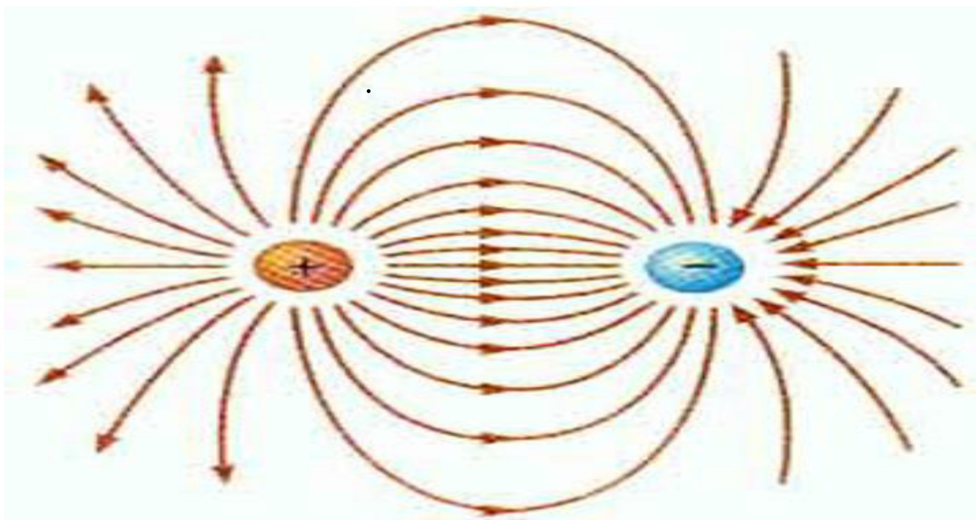
i) Isolated positive charge.



ii) Isolated negative charge



iii) Pair of equal and opposite charges (dipole)



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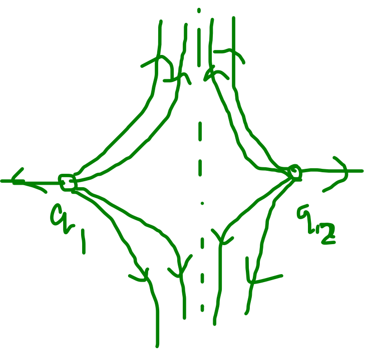
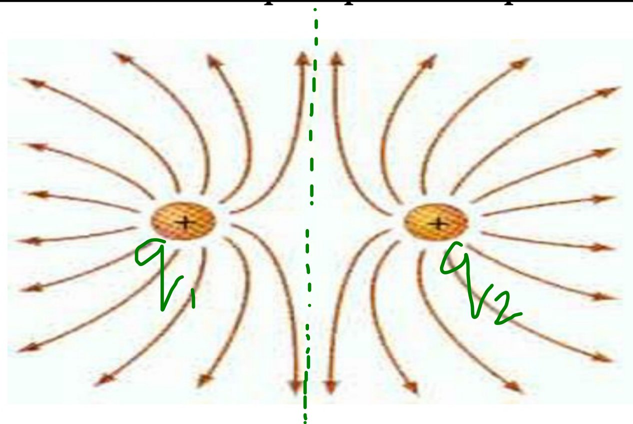
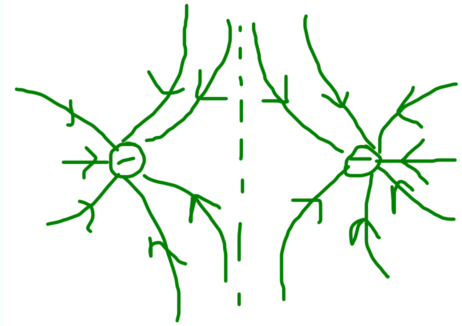
iv) <u>Line of force due to two equal positive point charges.</u>

$q_1 > q_2$

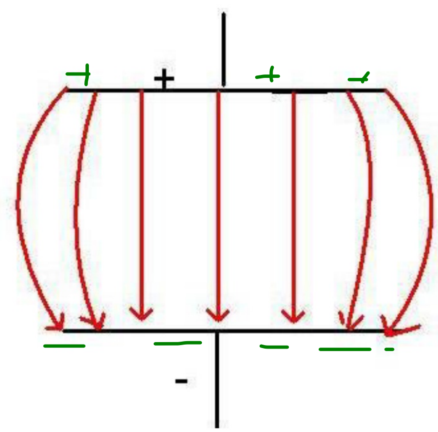


$q_1 = q_2$

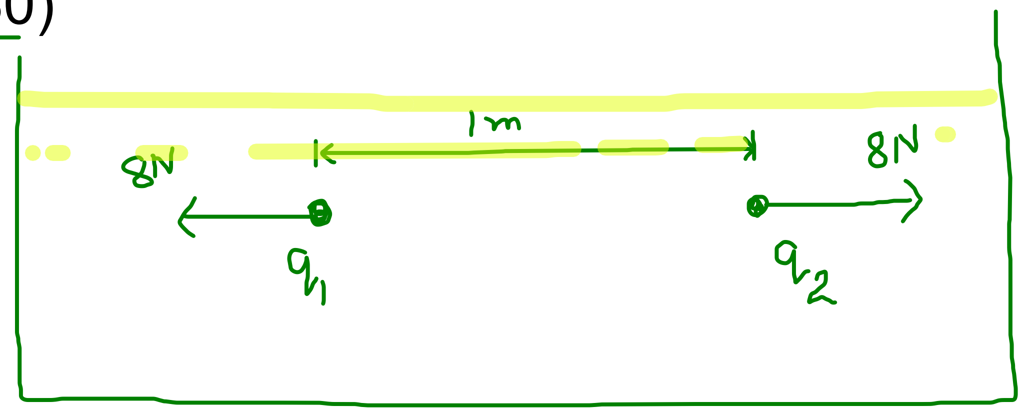




v) <u>Line of force due to two oppositely charges parallel plates.</u>



Que 10: Two-point charges having equal charges separated by 1m distance experience a force of 8N. What will be the force experienced by them, if they are held in water, at the same distance? (given Kwater=80)



Sol

$$K = \frac{\epsilon}{\epsilon_0} = \frac{F_v}{F_m}$$

given, $F_v = 8N$

$K_w = 80$

$$F_w = \frac{F_v}{K_w} = \_\_\_\_\_ ?$$

Que 11: Four-point charges $Q_A = 2\mu C$, $Q_B = -5\mu C$, $Q_C = 2\mu C$ and $Q_D = -5\mu C$ are located at the corners of a square ABCD of side 10 cm. What is the force on a charge of 1μC placed at the centre of the square?

$$\vec{F}_{net} = \vec{F}_{OA} + \vec{F}_{OB} + \vec{F}_{OC} + \vec{F}_{OD}$$

$$\vec{F}_{OA} = \frac{1}{4\pi\epsilon_0}\frac{Q_A q_0}{(AO)^2}\widehat{AO}$$

$$\vec{F}_{OB} = \frac{1}{4\pi\epsilon_0}\frac{Q_B q_0}{OB^2}\widehat{OB}$$

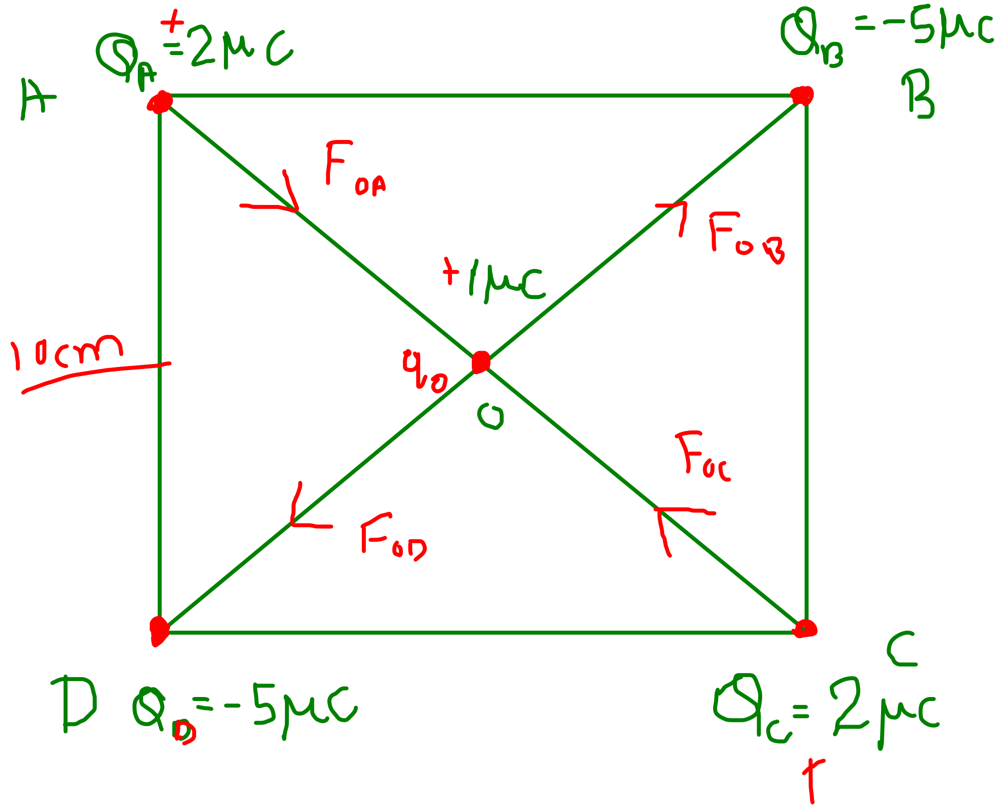
$$\vec{F}_{OC} = \frac{1}{4\pi\epsilon_0}\frac{Q_C q_0}{CO^2}\widehat{CO}$$

$$\vec{F}_{OD} = \frac{1}{4\pi\epsilon_0}\frac{Q_D q_0}{OD^2}\widehat{OD}$$

and, $AO = OB = CO = OD = r$

but, $\widehat{AO} = -\widehat{CO}$

$\widehat{OB} = -\widehat{OD}$



$= \boxed{\text{Zero}}$

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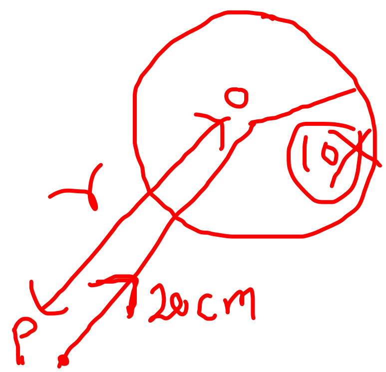
Que 12: A conducting sphere of radius 10 cm has an unknown charge. If the electric field 20 cm from the center of sphere is 1.5×10^{3} N/C and points radially inwards, then what is the net charge on the sphere?

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$1.5\times 10^{3} = 9\times 10^{9}\,\frac{q}{(20\times 10^{-2})^2}$$

$$q = ($$

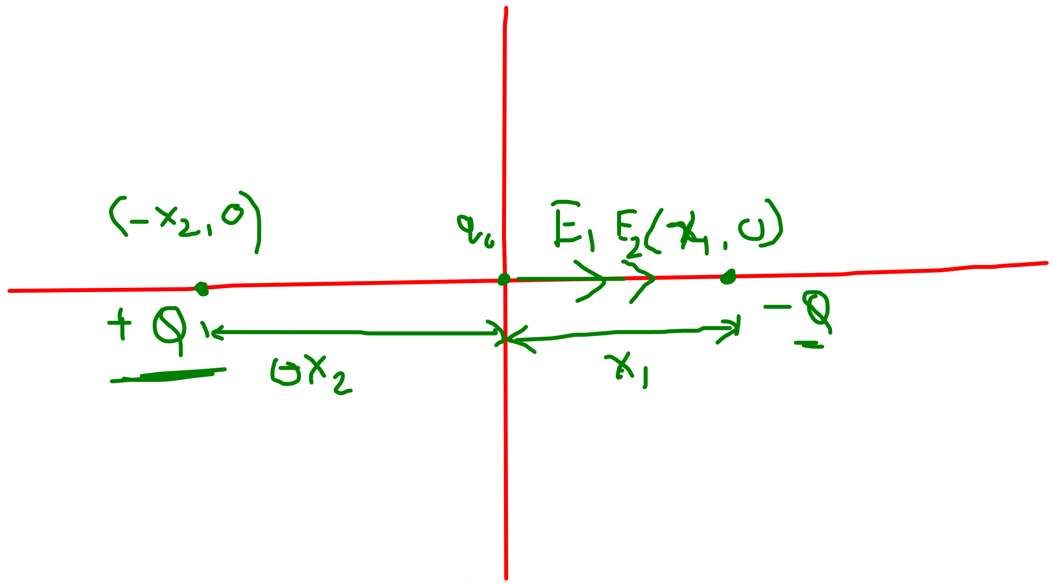


$\vec{E} = 1.5\times 10^{3}$ N/C

Que 13: Two charges +Q and -Q are kept at points ($-\mathbf{X}_2$, 0) and ($\mathbf{X}_1$,0) respectively, in the XY- plane. Find the magnitude and direction of the electric field at the origin (0,0).

$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{+Q}{x_2^2}$$

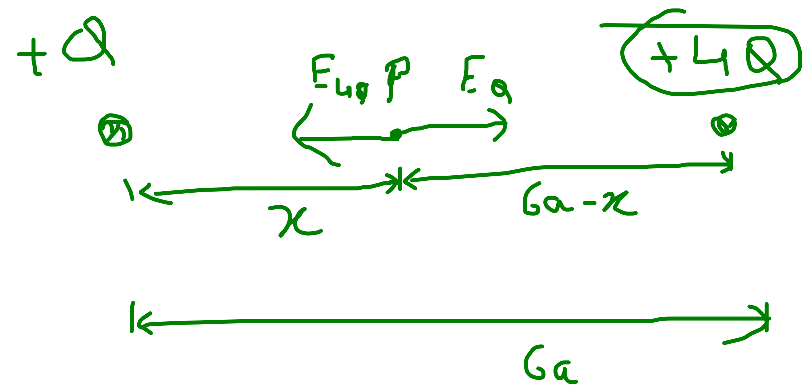
$$E_2 = \frac{1}{4\pi\epsilon_0} \frac{Q}{x_1^2}$$



$$E = E_1 + E_2$$

$$\boxed{\Rightarrow E = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{x_2^2} + \frac{1}{x_1^2}\right]}$$

Que 14: Two-point charges +Q and +4Q are separated by a distance of 6a. Find the point on the line joining the two charges, where the electric field is zero.



for electric field to be zero

$$E_{4Q} = E_Q$$

$$\cancel{\frac{1}{4\pi\epsilon_0}} \frac{4\cancel{Q}}{(6a-x)^2} = \cancel{\frac{1}{4\pi\epsilon_0}} \frac{\cancel{Q}}{x^2}$$

$$\boxed{\frac{4}{(6a-x)^2} = \frac{1}{x^2}}$$

$$x = 2a$$

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