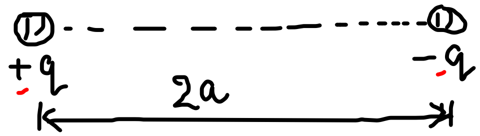
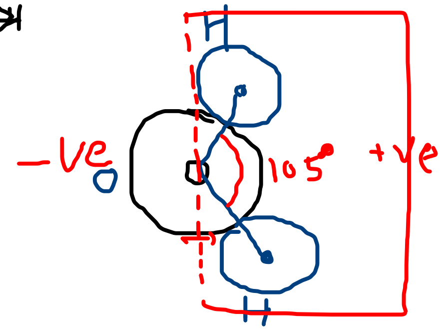
Electrostatics

Dipole: An electric dipole consists of a pair of equal and opposite point charges separated by a very small distance.

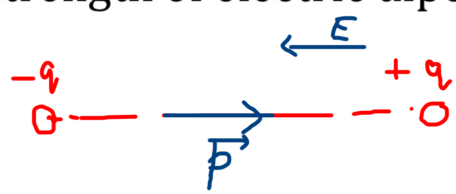


Example: H_2O



Dipole Moment ($\vec{p}$)

Dipole moment is the measure of the strength of electric dipole. It is vector quantity. Its magnitude is given by



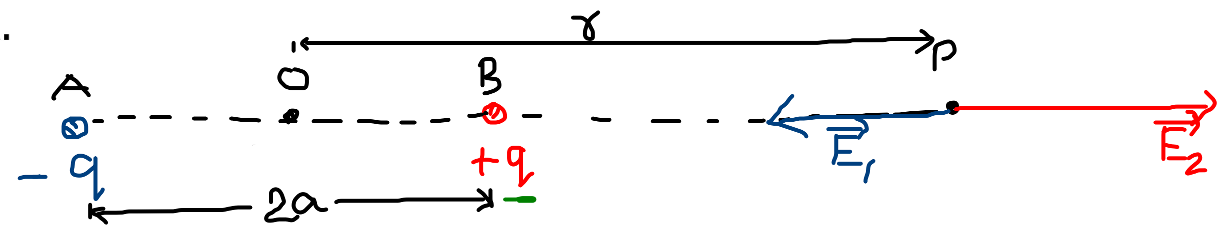
$$\vec{p} = q \times \vec{2a}$$

$$\text{Or, } |\vec{p}| = q \times |\vec{2a}|$$

The direction of $\vec{p}$ is from negative (-) to positive (+).

Electric field intensity at a point on axial line of electric dipole

Let us consider a dipole consisting of two point charges -q and +q separated by a small distance 2a.



We have to find out electric field at point P having distance r from O, on axial line of dipole.

If $\overrightarrow{E_1}$ is the electric intensity at P due to charge -q at A then,

$$\left|\overrightarrow{E_1}\right| = \frac{1}{4\pi\epsilon_0}\frac{q}{AP^2} = \frac{1}{4\pi\epsilon_0}\frac{q}{(r+a)^2} \text{ along PA}$$

and, if $\overrightarrow{E_2}$ at P due to charge +q at B then,

$$\left|\overrightarrow{E_2}\right| = \frac{1}{4\pi\epsilon_0}\frac{q}{\mathrm{BP}^2} = \frac{1}{4\pi\epsilon_0}\frac{q}{(r-a)^2} \text{ along BP}$$

as E_1 and E_2 are collinear vectors acting in opposite direction

and $\left|\overrightarrow{E_2}\right| > \left|\overrightarrow{E_1}\right|$

therefore, the resultant intensity E at P will be difference of two, acting along BP.

$$|\vec{E}| = |\vec{E_2}| - |\vec{E_1}|$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right] \qquad = \frac{q}{4\pi\epsilon_0}\left[\frac{(r+a)^2-(r-a)^2}{(r^2-a^2)^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{4ar}{(r^2-a^2)^2}\right]$$

$$|\vec{E}| = \frac{q\times 2a\times 2\times r}{4\pi\epsilon_0(r^2-a^2)^2}$$

$$|\vec{E}| = \frac{|\vec{p}|\times 2r}{4\pi\epsilon_0(r^2-a^2)^2} \qquad (\because q\times 2a = |\vec{p}|)$$

For short dipole i.e. 2a<< r

Then, a^2 will be negligible

so,

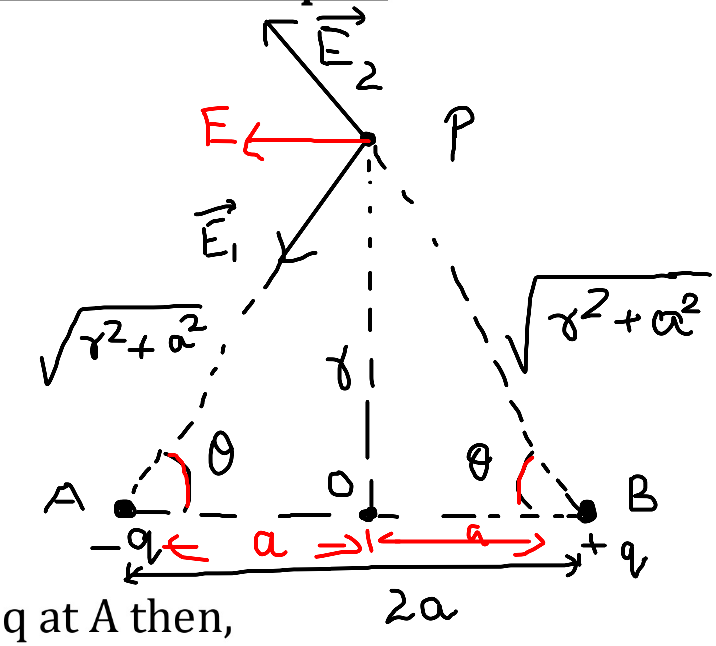
$$|\vec{E}| = \frac{|\vec{p}|\times 2r}{4\pi\epsilon_0 r^4}$$

$$|\vec{E}| = \frac{2|\vec{p}|}{4\pi\epsilon_0 r^3}$$

In Vectors, $\vec{E}_{axial} = \frac{2\vec{p}}{4\pi\epsilon_0 r^3}$

Electric field intensity at a point on equatorial line of electric dipole

Let us consider a dipole consisting of two point charges -q and +q separated by a small distance 2a. We have to find out electric field at point P having distance r from O, on equatorial line of dipole

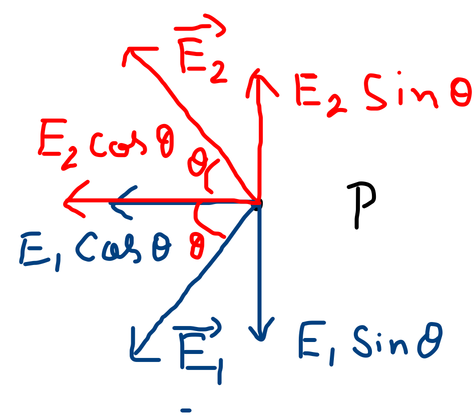


If $\vec{E_1}$ is the electric intensity at P due to charge -q at A then,

$$|\vec{E_1}| = \frac{1}{4\pi\epsilon_0}\frac{q}{AP^2} = \frac{1}{4\pi\epsilon_0}\frac{q}{\left(\sqrt{r+a}\right)^2} \text{ along PA}$$

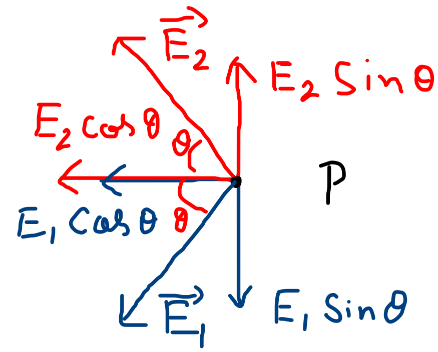
and, if $\vec{E_2}$ at P due to charge +q at B then,

$$|\vec{E_2}| = \frac{1}{4\pi\epsilon_0}\frac{q}{BP^2} = \frac{1}{4\pi\epsilon_0}\frac{q}{\left(\sqrt{r+a}\right)^2} \text{ along BP}$$



Magnitude of $\overrightarrow{E_1}$ and $\overrightarrow{E_2}$ are equal.

but $E_1 sin\theta$ and $E_2 sin\theta$ are in opposite direction, so they will cancel each other.



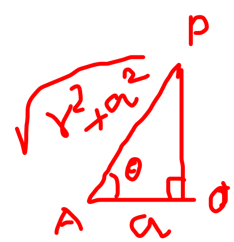
$$\therefore \left|\vec{E}\right| = E_1 \cos\theta + E_2 \cos\theta$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q\ cos\ \theta}{(r^2+a^2)} + \frac{1}{4\pi\epsilon_0}\frac{q\ cos\ \theta}{(r^2+a^2)}$$

$$= \frac{2q}{4\pi\epsilon_0}\frac{cos\ \theta}{(r^2+a^2)}$$

$$= \frac{2q}{4\pi\epsilon_0(r^2+a^2)}\left(\frac{\mathrm{BO}}{\mathrm{BP}}\right) = \frac{2q \times a}{4\pi\epsilon_0 (r^2+a^2)^1 (r^2+a^2)^{1/2}}$$

$$\left|\vec{E}\right| = \frac{2q\ a}{4\pi\epsilon_0(r^2+a^2)^{3\backslash 2}}$$



But, 2q X a = p

$$\left|\vec{E}\right| = \frac{|\vec{p}|}{4\pi\epsilon_0(r^2+a^2)^{3\backslash 2}}$$

$$\cos\theta = \frac{a}{\sqrt{r^2+a^2}}$$

For short dipole, 2a<< r, a^2 may be neglected

$$|\vec{E}| = \frac{|\vec{p}|}{4\pi\epsilon_0 (r^2)^{3/2}}$$

$$\text{Or, } |\vec{E}| = \frac{|\vec{p}|}{4\pi\epsilon_0 r^3}$$

$$\text{In vectors, } \vec{E}_{\text{equitorial}} = \frac{-\vec{p}}{4\pi\epsilon_0 r^3}$$

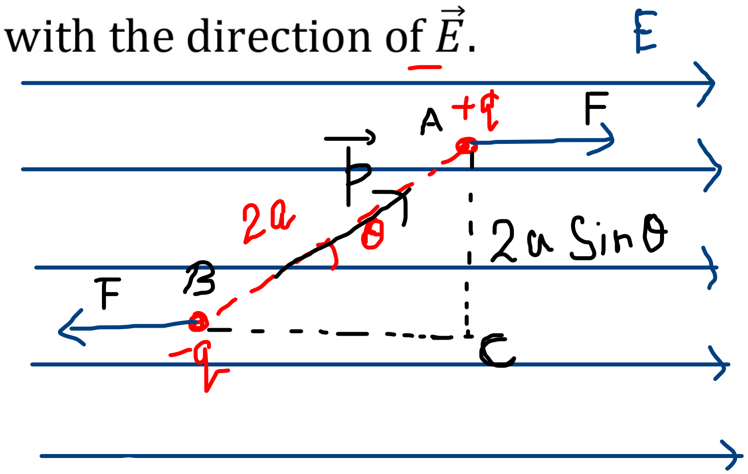
Negative sign shows that $\vec{E}$ is antiparallel with $\vec{p}$.

Note: For short dipole the intensity of the electric field on an axial point is double in magnitude by opposite in direction.

$$\vec{E}_{axial} = -\,2\vec{E}_{equitorial}$$

Torque on a dipole in a uniform electric field

Let us consider an electric dipole is placed in a uniform external electric field $\vec{E}$ at any angle θ with the direction of $\vec{E}$.



$\frac{p}{h} = \sin\theta$

$E = \frac{F}{q_0}$

Force on charge -q at B = q$\vec{E}$, along the direction opposite to $\vec{E}$

Force on charge +q at A = q$\vec{E}$, along the direction $\vec{E}$

These forces produce a couple which rotates the dipole in clockwise direction, $torque|\vec{\tau}| = moment\ of\ the\ couple$

= Force X arm of couple

= F X AC (perpendicular distance between the forces)

= q $|\vec{E}|$ X AB sin θ

$$= q\left|\vec{E}\right| \times 2a \sin\theta$$

$$= (q \times 2a)\left|\vec{E}\right| \sin\theta$$

$$|\vec{\tau}| = |\vec{p}|\left|\vec{E}\right| Sin\,\theta$$

$$In\ Vectors,\ \vec{\tau} = \vec{p} \times \vec{E}$$

$direction\ of\ \vec{\tau}\ is\ perpendicular\ to\ \vec{p}\ and\ \vec{E}$

Special Cases

i) When $\boldsymbol{\theta = 0}$, and $|\vec{\tau}| = |\vec{p}|\left|\vec{E}\right| Sin\,\theta$

$\sin 0^o = 0$

$\therefore\ \tau = 0$

i.e. when dipole is in the direction of field it is **stable equilibrium**.

But, for $\boldsymbol{\theta = 180^o}$ i.e. dipole is in opposite direction of $\vec{E}$
the torque τ would turn the dipole through 180^o,
and it will be in **unstable equilibrium**.

ii) When, $\boldsymbol{\theta = 90^o}$, $\sin 90^o = 1$

$\tau = pE$ ($Maximum$).