DON BOSCO SCHOOL, RANCHI

CLASS 9 SUBJECT: MATHEMATICS

Chapter 1 - Rational and Irrational Numbers

Exercise Ex. 1(A)

Question 1

Is zero a rational number? Can it be written in the form $\frac{p}{q}$, where p and q are integers and q≠0?

Solution 1

Yes, zero is a rational number.

As it can be written in the form of $\frac{p}{q}$, where p and q are integers and q≠0?

$\Rightarrow 0 = \frac{0}{1}$

Question 2

Are the following statements true or false? Give reasons for your answers.
i. Every whole number is a natural number.
ii. Every whole number is a rational number.
iii. Every integer is a rational number.
iv. Every rational number is a whole number.

Solution 2

i. False, zero is a whole number but not a natural number.

ii. True, Every whole can be written in the form of $\frac{p}{q}$, where p and q are integers and q≠0.

iii. True, Every integer can be written in the form of $\frac{p}{q}$, where p and q are integers and q≠0.

iv. False.

Example: $\frac{-2}{3}$ is a rational number, but not a whole number.

Question 3

Arrange $-\frac{5}{9}, \frac{7}{12}, -\frac{2}{3}$ and $\frac{11}{18}$ in ascending order of their magnitudes.
Also, find the difference between the largest and smallest of these fractions. Express this difference as a decimal fraction correct to one decimal place.

Solution 3

Consider the given numbers: $-\frac{5}{9}, \frac{7}{12}, -\frac{2}{3}$ and $\frac{11}{18}$

The L.C.M of 9,12, and 18 is 36

Thus the given numbers are:

$$-\frac{5}{9}, \frac{7}{12}, -\frac{2}{3} \text{ and } \frac{11}{18} = -\frac{5\times 4}{9\times 4}, \frac{7\times 3}{12\times 3}, -\frac{2\times 12}{3\times 12} \text{ and } \frac{11\times 2}{18\times 2}$$

$$= -\frac{20}{36}, \frac{21}{36}, -\frac{24}{36} \text{ and } \frac{22}{36}$$

Thus the numbers in ascending order are shown below:

$$-\frac{24}{36}, -\frac{20}{36}, \frac{21}{36} \text{ and } \frac{22}{36}$$

Thus the given numbers in ascending order are shown below:

$$-\frac{2}{3}, -\frac{5}{9}, \frac{7}{12} \text{ and } \frac{11}{18}$$

We need to find the difference between the largest and smallest of the above numbers.

$$\text{Thus, difference} = \frac{11}{18} - \left(-\frac{2}{3}\right)$$

$$= \frac{11}{18} + \frac{2}{3}$$

$$= \frac{11}{18} + \frac{2\times 6}{3\times 6}$$

$$= \frac{11}{18} + \frac{12}{18}$$

$$= \frac{11+12}{18}$$

$$= \frac{23}{18}$$

We need to express this fraction as a decimal, correct to one decimal place.

Thus, we have $\frac{23}{18} = 1.2\overline{7} \approx 1.3$.

Question 4

Arrange $\frac{5}{8}, -\frac{3}{16}, -\frac{1}{4}$ and $\frac{17}{32}$ in descending order of their magnitudes.

Also, find the sum of the lowest and largest of these fractions. Express the result obtained as a decimal fraction correct to two decimal places.

Solution 4

Consider the given numbers: $\frac{5}{8}, -\frac{3}{16}, -\frac{1}{4}$ and $\frac{17}{32}$.

The LCM of 8, 16, 4 and 32 is 32.

Thus, the given numbers are given below:

$$\frac{5}{8}, -\frac{3}{16}, -\frac{1}{4} \text{ and } \frac{17}{32} = \frac{5\times 4}{8\times 4}, -\frac{3\times 2}{16\times 2}, -\frac{1\times 8}{4\times 8} \text{ and } \frac{17\times 1}{32\times 1}$$

$$= \frac{20}{32}, -\frac{6}{32}, -\frac{8}{32} \text{ and } \frac{17}{32}$$

Thus, the numbers in descending order are shown below:

$\frac{20}{32}, \frac{17}{32}, -\frac{6}{32}$ and $-\frac{8}{32}$.

Thus, the given numbers in descending order are listed below:

$\frac{5}{8}, \frac{17}{32}, -\frac{3}{16}$ and $-\frac{1}{4}$.

We need to find the sum of the

largest and the smallest of the above numbers.

$$\text{Thus, sum} = \frac{5}{8} + \left(-\frac{1}{4}\right)$$

$$= \frac{5}{8} - \frac{1}{4}$$

$$= \frac{5}{8} - \frac{1\times 2}{4\times 2}$$

$$= \frac{5}{8} - \frac{2}{8}$$

$$= \frac{3}{8}$$

We need to express this fraction as a decimal,

correct to two decimal places.

Thus, we have $\frac{3}{8} = 0.375 \approx 0.38$.

Question 5

Without doing any actual division, find which of the following rational numbers have terminating decimal representation:

(i) $\frac{7}{16}$

(ii) $\frac{23}{125}$

(iii) $\frac{9}{14}$

(iv) $\frac{32}{45}$

(v) $\frac{43}{50}$

(vi) $\frac{17}{40}$

(vii) $\frac{61}{75}$

(viii) $\frac{123}{250}$

Solution 5

(i)

Given number is $\frac{7}{16}$

Since $16 = 2 \times 2 \times 2 \times 2 = 2^4 = 2^4 \times 5^0$

i.e. 16 can be expressed as $2^m \times 5^n$

$\therefore \frac{7}{16}$ is convertible into the terminating decimal.

(ii)

Given number is $\frac{23}{125}$

Since $125 = 5 \times 5 \times 5 = 5^3 = 2^0 \times 5^3$

i.e. 125 can be expressed as $2^m \times 5^n$

$\therefore \frac{23}{125}$ is convertible into the terminating decimal.

(iii)

Given number is $\frac{9}{14}$

Since $14 = 2 \times 7 = 2^1 \times 7^1$

i.e. 14 cannot be expressed as $2^m \times 5^n$

$\therefore \frac{9}{14}$ is not convertible into the terminating decimal.

(iv)

Given number is $\frac{32}{45}$

Since $45 = 3 \times 3 \times 5 = 3^2 \times 5^1$

i.e. 45 cannot be expressed as $2^m \times 5^n$

$\therefore \frac{32}{45}$ is not convertible into the terminating decimal.

(v)

Given number is $\frac{43}{50}$

Since $50 = 2 \times 5 \times 5 = 2^1 \times 5^2$

i.e. 50 can be expressed as $2^m \times 5^n$

$\therefore \frac{43}{50}$ is convertible into the terminating decimal.

(vi)

Given number is $\frac{17}{40}$

Since $40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5^1$

i.e. 40 can be expressed as $2^m \times 5^n$

$\therefore \frac{17}{40}$ is convertible into the terminating decimal.

(vii)

Given number is $\frac{61}{75}$

Since $75 = 3 \times 5 \times 5 = 3^1 \times 5^2$

i.e. 75 cannot be expressed as $2^m \times 5^n$

$\therefore \frac{61}{75}$ is not convertible into the terminating decimal.

(viii)

Given number is $\frac{123}{250}$

Since $250 = 2 \times 5 \times 5 \times 5 = 2^1 \times 5^3$

i.e. 250 can be expressed as $2^m \times 5^n$

$\therefore \frac{123}{250}$ is convertible into the terminating decimal.

Chapter 1 - Rational and Irrational Numbers

Exercise Ex. 1(B)

Question 1

State, whether the following numbers are rational or not:

(i) $\left(2+\sqrt{2}\right)^2$ (ii) $\left(3-\sqrt{3}\right)^2$ (iii) $\left(5+\sqrt{5}\right)\left(5-\sqrt{5}\right)$

(iv) $\left(\sqrt{3}-\sqrt{2}\right)^2$ (v) $\left(\frac{3}{2\sqrt{2}}\right)^2$ (vi) $\left(\frac{\sqrt{7}}{6\sqrt{2}}\right)^2$

Solution 1

(i) $(2+\sqrt{2})^2 = 2^2 + 2(2)(\sqrt{2}) + (\sqrt{2})^2$

$= 4 + 4\sqrt{2} + 2 = 6 + 4\sqrt{2}$

Irrational

(ii) $(3-\sqrt{3})^2 = (3)^2 - 2(3)(\sqrt{3}) + (\sqrt{3})^2$

$= 9 - 6\sqrt{3} + 3$

$= 12 - 6\sqrt{3} = 6(2-\sqrt{3})$

Irrational

(iii) $(5+\sqrt{5})(5-\sqrt{5}) = (5)^2 - (\sqrt{5})^2$

$= 25 - 5 = 20$

Rational

(iv) $(\sqrt{3}-\sqrt{2})^2 = (\sqrt{3})^2 - 2(\sqrt{3})(\sqrt{2}) + (\sqrt{2})^2$

$= 3 - 2\sqrt{6} + 2 = 5 - 2\sqrt{6}$ Irrational

(v) $\left(\frac{3}{2\sqrt{2}}\right)^2 = \frac{(3)^2}{(2\sqrt{2})^2} = \frac{9}{4\times 2} = \frac{9}{8}$ Rational

(vi) $\left(\frac{\sqrt{7}}{6\sqrt{2}}\right)^2 = \frac{(\sqrt{7})^2}{(6\sqrt{2})^2} = \frac{7}{36\times 2} = \frac{7}{72}$ Rational

Question 2

Find the square of:

$(i)\frac{3\sqrt{5}}{5}$ $(ii)\sqrt{3}+\sqrt{2}$ $(iii)\sqrt{5}-2$ $(iv)\,3+2\sqrt{5}$

Solution 2

(i)

$$\left(\frac{3\sqrt{5}}{5}\right)^2 = \frac{3^2(\sqrt{5})^2}{5^2}$$

$$= \frac{9\times 5}{25}$$

$$= \frac{9}{5}$$

$$= 1\frac{4}{5}$$

(ii)

$$\left(\sqrt{3}+\sqrt{2}\right)^2=\left(\sqrt{3}\right)^2+2\left(\sqrt{3}\right)\left(\sqrt{2}\right)+\left(\sqrt{2}\right)^2$$

$$=3+2\sqrt{6}+2=5+2\sqrt{6}$$

(iii)

$$(\sqrt{5}-2)^2=(\sqrt{5})^2-2(\sqrt{5})(2)+(2)^2$$

$$=5-4\sqrt{5}+4$$

$$=9-4\sqrt{5}$$

(iv)

$$(3+2\sqrt{5})^2=3^2+2(3)(2\sqrt{5})+(2\sqrt{5})^2$$

$$=9+12\sqrt{5}+20$$

$$=29+12\sqrt{5}$$

Question 3

State, in each case, whether true or false:

(i) $\sqrt{2}+\sqrt{3}=\sqrt{5}$

(ii) $2\sqrt{4}+2=6$

(iii) $3\sqrt{7}-2\sqrt{7}=\sqrt{7}$

(iv) $\frac{2}{7}$ is an irrational number

(v) $\frac{5}{11}$ is a rational number.

(vi) All rational numbers are real numbers.

(vii) All real numbers are rational numbers.

(viii) Some real numbers are rational numbers.

Solution 3

(i) False

(ii) $2\sqrt{4}+2=2\times 2+2=4+2=6$ which is true

(iii) $3\sqrt{7}-2\sqrt{7}=\sqrt{7}$ True.

(iv) False because

$$\frac{2}{7}=0.\overline{285714}$$

which is recurring and non-terminating and hence it is rational

(v) True because $\frac{5}{11}=0.\overline{45}$ which is recurring and non-terminating

(vi) True

(vii) False

(viii) True.

Question 4

Given universal set =

$$\left\{-6,\ -5\frac{3}{4},\ -\sqrt{4},\ -\frac{3}{5},\ -\frac{3}{8},\ 0,\ \frac{4}{5},\ 1,\ 1\frac{2}{3},\ \sqrt{8},\ 3.01,\ \pi,\ 8.47\right\}$$

From the given set, find :
(i) set of rational numbers
(ii) set of irrational numbers
(iii) set of integers
(iv) set of non-negative integers

Solution 4

Given Universal set is

$$\left\{-6,\ -5\frac{3}{4},\ -\sqrt{4},\ -\frac{3}{5},\ -\frac{3}{8},\ 0,\ \frac{4}{5},\ 1,\ 1\frac{2}{3},\ \sqrt{8},\ 3.01,\ \pi,\ 8.47\right\}$$

(i)

We need to find the set of rational numbers.

Rational numbers are numbers of the form $\frac{p}{q}$, where $q \neq 0$.

$$U = \left\{-6,\ -5\frac{3}{4},\ -\sqrt{4},\ -\frac{3}{5},\ -\frac{3}{8},\ 0,\ \frac{4}{5},\ 1,\ 1\frac{2}{3},\ \sqrt{8},\ 3.01,\ \pi,\ 8.47\right\}$$

Clearly, $-5\frac{3}{4}$, $-\frac{3}{5}$, $-\frac{3}{8}$, $\frac{4}{5}$ and $1\frac{2}{3}$ are of the form $\frac{p}{q}$.

Hence, they are rational numbers.

Since the set of integers is a subset of rational numbers,

-6, 0 and 1 are also rational numbers.

Thus, decimal numbers 3.01 and 8.47 are also rational numbers

because they are terminating decimals.

Hence, from the above set, the set of rational

numbers is Q, and $Q = \left\{-6,\ -5\frac{3}{4},\ -\frac{3}{5},\ -\frac{3}{8},\ 0,\ \frac{4}{5},\ 1,\ 1\frac{2}{3},\ 3.01,\ 8.47\right\}$

ii)

We need to find the set of irrational numbers.
Irrational numbers are numbers which are not rational.
From the above subpart, the set of rational
numbers is Q,
and $Q=\left\{-6,\ -5\frac{3}{4},\ -\frac{3}{5},\ -\frac{3}{8},\ 0,\ \frac{4}{5},\ 1,\ 1\frac{2}{3},\ 3.01,\ 8.47\right\}$
Set of irrational numbers is the set of complement of
the rational numbers over real numbers.
Here the set of irrational numbers is $U-Q=\left\{\sqrt{8},\pi\right\}$

(iii)

We need to find the set of integers.
Set of integers consists of zero, the natural numbers
and their additive inverses.
The set of integers is Z
$Z=\{\ldots-3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ldots\}$
Here the set of integers is $U\cap Z=\left\{-6,\ \sqrt{4},\ 0,\ 1\right\}$.

(iv)

We need to find the set of non-negative integers.
Set of non-negative integers consists of zero and
the natural numbers.
The set of non-negative integers is Z^+ and
$Z^+=\{0,1,2,3,\ldots\}$
Here the set of integers is $U\cap Z^+=\{0,1\}$

Question 5

Use method of contradiction to show that $\sqrt{3}$ and $\sqrt{5}$ are irrational numbers.

Solution 5

Let us suppose that $\sqrt{3}$ and $\sqrt{5}$ are rational numbers
$\therefore \sqrt{3}=\frac{a}{b}$ and $\sqrt{5}=\frac{x}{y}$ (Where a, b $\in$ 7 and b, y $\neq$ 0 x , y)
Squaring both sides
$3=\frac{a^2}{b^2}$, $5=\frac{x^2}{y^2}$

$3b^2 = a^2 \quad , 5y^2 = x^2 \} \quad \ldots(*)$

$\Rightarrow a^2$ and x^2 are odd as $3b^2$ and $5y^2$ are odd .

$\Rightarrow$ a and x are odd....(1)

Let a = 3c, x = 5z

$a^2 = 9c^2$, $x^2 = 25z^2$

$3b^2 = 9c^2$, $5y^2 = 25z^2$(From equation $(*)$)

$\Rightarrow b^2 = 3c^2$, $y^2 = 5z^2$

$\Rightarrow b^2$ and y^2 are odd as $3c^2$ and $5z^2$ are odd .

$\Rightarrow$ b and y are odd...(2)

From equation (1) and (2) we get a, b, x, y are odd integers.

i.e., a, b, and x, y have common factors 3 and 5 this contradicts our assumption that $\frac{a}{b}$ and $\frac{x}{y}$ are rational i.e, a, b and x, y do not have any common factors other than.

$\Rightarrow \frac{a}{b}$ and $\frac{x}{y}$ is not rational

$\Rightarrow \sqrt{3}$ and $\sqrt{5}$ are irrational.

Question 6

Prove that each of the following numbers is irrational:

i. $\sqrt{3} + \sqrt{2}$

ii. $3 - \sqrt{2}$

iii. $\sqrt{5} - 2$

Solution 6

i. $\sqrt{3} + \sqrt{2}$

Let $\sqrt{3} + \sqrt{2}$ be a rational number.

$\Rightarrow \sqrt{3} + \sqrt{2} = x$

Squaring on both the sides, we get

$$\left(\sqrt{3} + \sqrt{2}\right)^2 = x^2$$

$$\Rightarrow 3 + 2 + 2 \times \sqrt{3} \times \sqrt{2} = x^2$$

$$\Rightarrow x^2 - 5 = 2\sqrt{6}$$

$$\Rightarrow \sqrt{6} = \frac{x^2 - 5}{2}$$

Here, x is a rational number.

$\Rightarrow x^2$ is a rational number.

$\Rightarrow x^2 - 5$ is a rational number.

$\Rightarrow \frac{x^2 - 5}{2}$ is also a rational number.

$\Rightarrow \frac{x^2 - 5}{2} = \sqrt{6}$ is a rational number.

But $\sqrt{6}$ is an irrational number.

$\Rightarrow \frac{x^2 - 5}{2}$ is an irrational number.

$\Rightarrow x^2 - 5$ is an irrational number.

$\Rightarrow x^2$ is an irrational number.

$\Rightarrow$ x is an irrational number.

But we have assume that x is a rational number.

$\therefore$ we arrive at a contradiction.

So, our assumption that $\sqrt{3} + \sqrt{2}$ is a rational number is wrong.

$\therefore \sqrt{3} + \sqrt{2}$ is an irrational number.

ii. $3 - \sqrt{2}$

Let $3 - \sqrt{2}$ be a rational number.

$\Rightarrow 3 - \sqrt{2} = x$

Squaring on both the sides, we get

$$\left(3 - \sqrt{2}\right)^2 = x^2$$

$$\Rightarrow 9 + 2 - 2 \times 3 \times \sqrt{2} = x^2$$

$$\Rightarrow 11 - x^2 = 6\sqrt{2}$$

$$\Rightarrow \sqrt{2} = \frac{11 - x^2}{6}$$

Here, x is a rational number.

$\Rightarrow x^2$ is a rational number.

$\Rightarrow 11 - x^2$ is a rational number.

$\Rightarrow \frac{11 - x^2}{6}$ is also a rational number.

$\Rightarrow \sqrt{2} = \frac{11 - x^2}{6}$ is a rational number.

But $\sqrt{2}$ is an irrational number.

$\Rightarrow \frac{11 - x^2}{6} = \sqrt{2}$ is an irrational number.

$\Rightarrow 11 - x^2$ is an irrational number.

$\Rightarrow x^2$ is an irrational number.

$\Rightarrow$ x is an irrational number.

But we have assume that x is a rational number.

$\therefore$ we arrive at a contradiction.

So, our assumption that $3 - \sqrt{2}$ is a rational number is wrong.

$\therefore 3 - \sqrt{2}$ is an irrational number.

iii. $\sqrt{5} - 2$

Let $\sqrt{5} - 2$ be a rational number.

$\Rightarrow \sqrt{5} - 2 = x$

Squaring on both the sides, we get

$(\sqrt{5} - 2)^2 = x^2$

$\Rightarrow 5 + 4 - 2 \times 2 \times \sqrt{5} = x^2$

$\Rightarrow 9 - x^2 = 4\sqrt{5}$

$\Rightarrow \sqrt{5} = \frac{9 - x^2}{4}$

Here, x is a rational number.

$\Rightarrow x^2$ is a rational number.

$\Rightarrow 9 - x^2$ is a rational number.

$\Rightarrow \frac{9 - x^2}{4}$ is also a rational number.

$\Rightarrow \sqrt{2} = \frac{11 - x^2}{6}$ is a rational number.

But $\sqrt{2}$ is an irrational number.

$\Rightarrow \sqrt{5} = \frac{9 - x^2}{4}$ is an irrational number.

$\Rightarrow 9 - x^2$ is an irrational number.

$\Rightarrow x^2$ is an irrational number.

$\Rightarrow x$ is an irrational number.

But we have assume that x is a rational number.

$\therefore$ we arrive at a contradiction.

So, our assumption that $\sqrt{5} - 2$ is a rational number is wrong.

$\therefore \sqrt{5} - 2$ is an irrational number.

Question 7

Write a pair of irrational numbers whose sum is irrational.

Solution 7

$\sqrt{3}+5$ and $\sqrt{5}-3$ are irrational numbers whose sum is irrational.

$(\sqrt{3}+5)+(\sqrt{5}-3)=\sqrt{3}+\sqrt{5}+5-3=\sqrt{3}+\sqrt{5}+2$ which is irrational.

Question 8

Write a pair of irrational numbers whose sum is rational.

Solution 8

$\sqrt{3}+5$ and $4-\sqrt{3}$ are two irrational numbers whose sum is rational.

$(\sqrt{3}+5)+(4-\sqrt{3})=\sqrt{3}+5+4-\sqrt{3}=9$

Question 9

Write a pair of irrational numbers whose difference is irrational.

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Solution 9

$\sqrt{3}+2$ and $\sqrt{2}-3$ are two irrational numbers whose difference is irrational.

$(\sqrt{3}+2)-(\sqrt{2}-3)=\sqrt{3}+2-\sqrt{2}+3=\sqrt{3}-\sqrt{2}+5$ which is irrational.

Question 10

Write a pair of irrational numbers whose difference is rational.

Solution 10

$\sqrt{5}-3$ and $\sqrt{5}+3$ are irrational numbers whose difference is rational.

$(\sqrt{5}-3)-(\sqrt{5}+3)=\sqrt{5}-3-\sqrt{5}-3=-6$ which is rational.

Question 11

Write a pair of irrational numbers whose product is irrational.

Solution 11

Consider two irrational numbers $(5+\sqrt{2})$ *and* $(\sqrt{5}-2)$

Thus, the product, $(5+\sqrt{2})\times(\sqrt{5}-2)=5\sqrt{5}-10+\sqrt{10}-2\sqrt{2}$ *is irrational.*

Question 12

Write in ascending order:

(i) $3\sqrt{5}$ and $4\sqrt{3}$

(ii) $2\sqrt[3]{5}$ and $3\sqrt[3]{2}$

(iii) $6\sqrt{5}$, $7\sqrt{3}$ and $8\sqrt{2}$

Solution 12

(i) $3\sqrt{5}=\sqrt{3^2\times5}=\sqrt{45}, 4\sqrt{3}=\sqrt{4^2\times3}=\sqrt{48}$

and $45 < 48$ $\therefore \sqrt{45}<\sqrt{48} \Rightarrow 3\sqrt{5}<4\sqrt{3}$

(ii) $2\sqrt[3]{5}=\sqrt[3]{2^3\times5}=\sqrt[3]{40}, 3\sqrt[3]{2}=\sqrt[3]{3^3\times2}=\sqrt[3]{54}$

and $40 < 54$ $\Rightarrow \sqrt[3]{40}<\sqrt[3]{54}$

$\Rightarrow 2\sqrt[3]{5}<3\sqrt[3]{2}$

(iii) $6\sqrt{5}=\sqrt{6^2\times5}=\sqrt{180}$

$7\sqrt{3}=\sqrt{7^2\times3}=\sqrt{147}$

$8\sqrt{2}=\sqrt{8^2\times2}=\sqrt{128}$

and $128 < 147 < 180$

$\therefore \sqrt{128}<\sqrt{147}<\sqrt{180}$

$\Rightarrow 8\sqrt{2}<7\sqrt{3}<6\sqrt{5}$

Question 13

Write in ascending order:

(i) $3\sqrt{5}$ and $4\sqrt{3}$

(ii) $2\sqrt[3]{5}$ and $3\sqrt[3]{2}$

(iii) $6\sqrt{5}, 7\sqrt{3}$ and $8\sqrt{2}$

Question 13

Write in ascending order:

(i) $3\sqrt{5}$ and $4\sqrt{3}$

(ii) $2\sqrt[3]{5}$ and $3\sqrt[3]{2}$

(iii) $6\sqrt{5}, 7\sqrt{3}$ and $8\sqrt{2}$

Question 14

Write in descending order:

(i) $2\sqrt[4]{6}$ and $3\sqrt[4]{2}$

(ii) $7\sqrt{3}$ and $3\sqrt{7}$

Solution 14

(i) $2\sqrt[4]{6} = \sqrt[4]{2^4 \times 6} = \sqrt[4]{96}$

$3\sqrt[4]{2} = \sqrt[4]{3^4 \times 2} = \sqrt[4]{162}$

Since 162 > 96

$\Rightarrow \sqrt[4]{162} > \sqrt[4]{96}$

$\Rightarrow 3\sqrt[4]{2} > 2\sqrt[4]{6}$

(ii) $7\sqrt{3} = \sqrt{7^2 \times 3} = \sqrt{141}$

$3\sqrt{7} = \sqrt{3^2 \times 7} = \sqrt{63}$

$141 > 63 \Rightarrow \sqrt{141} > \sqrt{63}$

Question 15

Compare.

(i) $\sqrt[6]{15}$ and $\sqrt[4]{12}$

(ii) $\sqrt{24}$ and $\sqrt[3]{35}$

Solution 15

(i) $\sqrt[6]{15} = (15)^{\frac{1}{6}}$ and $\sqrt[4]{12} = (12)^{\frac{1}{4}}$

Make powers $\frac{1}{6}$ and $\frac{1}{4}$ same

L.C.M. of 6,4 is 12

$\frac{1}{6} \times \frac{2}{2} = \frac{2}{12}$

and $\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$

$\Rightarrow \sqrt[6]{15} = (15)^{\frac{1}{6}} = (15)^{\frac{2}{12}} = \left(15^2\right)^{\frac{1}{12}} = (225)^{\frac{1}{12}}$

$\text{and}\sqrt[4]{12} = (12)^{\frac{1}{4}} = (12)^{\frac{3}{12}} = \left(12^3\right)^{\frac{1}{12}} = (1728)^{\frac{1}{12}}$

$\Rightarrow 1272 > 225$

$\Rightarrow (1728)^{\frac{1}{12}} > (225)^{\frac{1}{12}}$

$\Rightarrow \sqrt[4]{12} > \sqrt[6]{15}$

(ii) $\sqrt{24} = (24)^{\frac{1}{2}}$ and $\sqrt[3]{35} = (35)^{\frac{1}{3}}$

L.C.M. of 2 and 3 is 6.

$\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}, \quad \frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$

$\Rightarrow (24)^{\frac{1}{2}} = (24)^{\frac{3}{6}} = \left(24^3\right)^{\frac{1}{6}} = (13824)^{\frac{1}{6}}$

$(35)^{\frac{1}{3}} = (35)^{\frac{2}{6}} = \left(35^2\right)^{\frac{1}{6}} = (1225)^{\frac{1}{6}}$

$\Rightarrow 13824 > 1225$

$\Rightarrow (13824)^{\frac{1}{6}} > \sqrt[3]{35}$

$\Rightarrow \sqrt{24} > \sqrt[3]{35}$

Question 16

Insert two irrational numbers between 5 and 6.

Solution 16

We know that $5 = \sqrt{25}$ *and* $6 = \sqrt{36}$.

Thus consider the numbers,

$\sqrt{25} < \sqrt{26} < \sqrt{27} < \sqrt{28} < \sqrt{29} < \sqrt{30} < \sqrt{31} < \sqrt{32} < \sqrt{33} < \sqrt{34} < \sqrt{35} < \sqrt{36}$

Therefore, any two irrational numbers between 5 *and* 6 *is* $\sqrt{27}$ *and* $\sqrt{28}$

Question 17

Insert five irrational numbers between $2\sqrt{5}$ and $3\sqrt{3}$.

Solution 17

We know that $2\sqrt{5} = \sqrt{4 \times 5} = \sqrt{20}$ *and* $3\sqrt{3} = \sqrt{27}$

Thus, we have, $\sqrt{20} < \sqrt{21} < \sqrt{22} < \sqrt{23} < \sqrt{24} < \sqrt{25} < \sqrt{26} < \sqrt{27}$

So any five irrational numbers between $2\sqrt{5}$ *and* $3\sqrt{3}$ *are:*

$\sqrt{21}, \sqrt{22}, \sqrt{23}, \sqrt{24}$ *and* $\sqrt{26}$

Question 18

Write two rational numbers between $\sqrt{2}$ and $\sqrt{3}$.

Solution 18

We want rational numbers a/b and c/d such that: $\sqrt{2} < a/b < c/d < \sqrt{3}$

Consider any two rational numbers between 2 and 3 such that they are perfect squares.

$$\sqrt{2.25} = 1.5 \text{ and } \sqrt{2.56} = 1.6$$

Let us take 2.25 and 2.56 as

Thus we have,

$$\sqrt{2} < \sqrt{2.25} < \sqrt{2.56} < \sqrt{3}$$

$$\Rightarrow \sqrt{2} < 1.5 < 1.6 < \sqrt{3}$$

$$\Rightarrow \sqrt{2} < \frac{15}{10} < \frac{16}{10} < \sqrt{3}$$

$$\Rightarrow \sqrt{2} < \frac{3}{2} < \frac{8}{5} < \sqrt{3}$$

Therefore any two rational numbers between $\sqrt{2}$ *and* $\sqrt{3}$ *are:* $\frac{3}{2}$ *and* $\frac{8}{5}$

Question 19

Write three rational numbers between $\sqrt{3}$ and $\sqrt{5}$.

Solution 19

Consider some rational numbers between 3 and 5 such that they are perfect squares.

Let us take, 3.24, 3.61, 4, 4.41 and 4.84 as

$$\sqrt{3.24} = 1.8, \sqrt{3.61} = 1.9, \sqrt{4} = 2, \sqrt{4.41} = 2.1 \text{ and } \sqrt{4.84} = 2.2$$

Thus we have,

$$\sqrt{3} < \sqrt{3.24} < \sqrt{3.61} < \sqrt{4} < \sqrt{4.41} < \sqrt{4.84} < \sqrt{5}$$

$$\Rightarrow \sqrt{3} < 1.8 < 1.9 < 2 < 2.1 < 2.2 < \sqrt{5}$$

$$\Rightarrow \sqrt{3} < \frac{18}{10} < \frac{19}{10} < 2 < \frac{21}{10} < \frac{22}{10} < \sqrt{5}$$

$$\Rightarrow \sqrt{3} < \frac{9}{5} < \frac{19}{10} < 2 < \frac{21}{10} < \frac{11}{5} < \sqrt{5}$$

Therefore, any three rational numbers between $\sqrt{3}$ *and* $\sqrt{5}$ *are:*

$\frac{9}{5}, \frac{19}{10}$ *and* $\frac{21}{10}$

Question 20

Simplify each of the following:

(i) $\sqrt[5]{16} \times \sqrt[5]{2}$

(ii) $\frac{\sqrt[4]{243}}{\sqrt[4]{3}}$

(iii) $(3+\sqrt{2})(4+\sqrt{7})$

(iv) $(\sqrt{3}-\sqrt{2})^2$

Solution 20

(i)

$$\sqrt[5]{16} \times \sqrt[5]{2}$$

$$= 16^{\frac{1}{5}} \times 2^{\frac{1}{5}}$$

$$= 2^{4\times\frac{1}{5}} \times 2^{\frac{1}{5}}$$

$$= 2^{\frac{4}{5}} \times 2^{\frac{1}{5}}$$

$$= 2^{\frac{4}{5}+\frac{1}{5}}$$

$$= 2^{\frac{5}{5}}$$

$$= 2^1$$

$$= 2$$

(ii)

$$\frac{\sqrt[4]{243}}{\sqrt[4]{3}}$$

$$= \frac{\sqrt[4]{3^5}}{\sqrt[4]{3}}$$

$$= \frac{3^{5\times\frac{1}{4}}}{3^{\frac{1}{4}}}$$

$$= \frac{3^{\frac{5}{4}}}{3^{\frac{1}{4}}}$$

$$= 3^{\frac{5}{4}-\frac{1}{4}}$$

$$= 3^{\frac{4}{4}}$$

$$= 3$$

(iii)

$(3+\sqrt{2})(4+\sqrt{7})$

$= 3\times 4+3\times\sqrt{7}+4\times\sqrt{2}+\sqrt{2}\times\sqrt{7}$

$= 12+3\sqrt{7}+4\sqrt{2}+\sqrt{14}$

(iv)

$(\sqrt{3}-\sqrt{2})^2$

$= (\sqrt{3})^2+(\sqrt{2})^2-2\times\sqrt{3}\times\sqrt{2}$

$= 3+2-2\sqrt{6}$

$= 5-2\sqrt{6}$

Chapter 1 - Rational and Irrational Numbers

Exercise Ex. 1(C)

Question 1

State, with reasons, which of the following are surds and which are not:

(i) $\sqrt{180}$

(ii) $\sqrt[4]{27}$

(iii) $\sqrt[5]{128}$

(iv) $\sqrt[3]{64}$

(v) $\sqrt[3]{25}.\sqrt[3]{40}$

(vi) $\sqrt[3]{-125}$

(vii) $\sqrt{\pi}$

(viii) $\sqrt{3+\sqrt{2}}$

Solution 1

(i) $\sqrt{180}=\sqrt{2\times 2\times 5\times 3\times 3}=6\sqrt{5}$ Which is irrational

$\therefore \sqrt{180}$ is a surds

(ii) $\sqrt[4]{27}=\sqrt[4]{3\times 3\times 3}$ Which is irrational

$\therefore \sqrt[4]{24}$ is a surds

(iii) $\sqrt[5]{128}=\sqrt[5]{2\times 2\times 2\times 2\times 2\times 2\times 2}=2\sqrt[5]{4}$

$\therefore \sqrt[5]{128}$ is a surds

(iv) $\sqrt[3]{64} = \sqrt[3]{2\times 2\times 2\times 2\times 2\times 2} = 4$ which is rational

$\therefore \sqrt[3]{34}$ is not a surds

(v) $\sqrt[3]{25}.\sqrt[3]{40} = \sqrt[3]{5\times 5\times 2\times 2\times 2\times 5} = 2\times 5 = 10$

$\therefore \sqrt[3]{25}.\sqrt[3]{40}$ is not a surds

(vi) $\sqrt[3]{-125} = \sqrt[3]{-5\times -5\times -5} =$ -5

$\therefore$ is is not a surds

(vii) $\sqrt{\pi}$ not a surds as π is irrational

(viii) $\sqrt{3+\sqrt{2}}$ is not a surds because $3+\sqrt{2}$ is irrational.

Question 2

Write the lowest rationalising factor of:

(i) $5\sqrt{2}$

(ii) $\sqrt{24}$

(iii) $\sqrt{5}-3$

(iv) $7-\sqrt{7}$

(v) $\sqrt{18}-\sqrt{50}$

(vi) $\sqrt{5}-\sqrt{2}$

(vii) $\sqrt{13}+3$

(viii) $15-3\sqrt{2}$

(ix) $3\sqrt{2}+2\sqrt{3}$

Solution 2

(i) $5\sqrt{2}\times\sqrt{2} = 5\times 2 = 10$ which is rational

$\therefore$ lowest rationalizing factor is $\sqrt{2}$

(ii) $\sqrt{24} = \sqrt{2\times 2\times 2\times 3} = 2\sqrt{6}$

$\therefore$ lowest rationalizing factor is $\sqrt{6}$

(iii) $\left(\sqrt{5}-3\right)\left(\sqrt{5}+3\right) = \left(\sqrt{5}\right)^2 - (3)^2 = 5-9 = -4$

$\therefore$ lowest rationalizing factor is $\left(\sqrt{5}+3\right)$

(iv) $7-\sqrt{7}$

$(7-\sqrt{7})(7+\sqrt{7})=49-7=42$

Therefore, lowest rationalizing factor is $(7+\sqrt{7})$

(v) $\sqrt{18}-\sqrt{50}$

$\sqrt{18}-\sqrt{50}=\sqrt{2\times3\times3}-\sqrt{5\times5\times2}$

$=3\sqrt{2}-5\sqrt{2}=-2\sqrt{2}$

$\therefore$ lowest rationalizing factor is $\sqrt{2}$

(vi) $\sqrt{5}-\sqrt{2}$

$(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})=(\sqrt{5})^2-(\sqrt{2})^2=3$

Therefore lowest rationalizing factor is $\sqrt{5}+\sqrt{2}$

(vii) $\sqrt{13}+3$

$(\sqrt{13}+3)(\sqrt{13}-3)=(\sqrt{13})^2-3^2=13-9=4$

Its lowest rationalizing factor is $\sqrt{13}-3$

(viii) $15-3\sqrt{2}$

$15-3\sqrt{2}=3(5-\sqrt{2})$

$=3(5-\sqrt{2})(5+\sqrt{2})$

$=3\times\left[5^2-(\sqrt{2})^2\right]$

$=3\times[25-2]$

$=3\times23$

$=69$

Its lowest rationalizing factor is $5+\sqrt{2}$

(ix) $3\sqrt{2}+2\sqrt{3}$

$$3\sqrt{2}+2\sqrt{3}=(3\sqrt{2}+2\sqrt{3})(3\sqrt{2}-2\sqrt{3})$$
$$=(3\sqrt{2})^2-(2\sqrt{3})^2$$
$$=9\times 2-4\times 3$$
$$=18-12$$
$$=6$$

its lowest rationalizing factor is $3\sqrt{2}-2\sqrt{3}$

Question 3

Rationalise the denominators of :

(i) $\frac{3}{\sqrt{5}}$

(ii) $\frac{2\sqrt{3}}{\sqrt{5}}$

(iii) $\frac{1}{\sqrt{3}-\sqrt{2}}$

(iv) $\frac{3}{\sqrt{5}+\sqrt{2}}$

(v) $\frac{2-\sqrt{3}}{2+\sqrt{3}}$

(vi) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

(vii) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

(viii) $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}$

(ix) $\frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}-3\sqrt{2}}$

Solution 3

(i)

$$\frac{3}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{3\sqrt{5}}{5}$$

(ii)

$$\frac{2\sqrt{3}}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{2}{5}\sqrt{15}$$

(iii)

$$\frac{1}{\sqrt{3}-\sqrt{2}}\times\left(\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right)=\frac{\sqrt{3+\sqrt{2}}}{(\sqrt{3})^2-(\sqrt{2})^2}=\frac{\sqrt{3}+\sqrt{2}}{3-2}$$

$$=\sqrt{3}+\sqrt{2}$$

(iv)

$$\frac{3}{\sqrt{5}+\sqrt{2}}\times\left(\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}\right)=\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}=\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{5-2}$$

$$=\sqrt{5}-\sqrt{2}$$

(v)

$$\frac{2-\sqrt{3}}{2+\sqrt{3}}\times\left(\frac{2-\sqrt{3}}{2-\sqrt{3}}\right)=\frac{\left(2-\sqrt{3}\right)^2}{(2)^2-\left(\sqrt{3}\right)^2}=\frac{4+3-4\sqrt{3}}{4-3}$$

$$=\frac{7-4\sqrt{3}}{1}=7-4\sqrt{3}$$

(vi)

$$\frac{\sqrt{3}+1}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}=\frac{\left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}\right)^2-(1)^2}=\frac{3+1+2\sqrt{3}}{3-1}=\frac{4+2\sqrt{3}}{2}$$

$$=\frac{\cancel{2}\left(2+\sqrt{3}\right)}{\cancel{2}}=2+\sqrt{3}$$

(vii)

$$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}=\frac{\left(\sqrt{3}-\sqrt{2}\right)^2}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}=\frac{3+2-2\sqrt{6}}{3-2}$$

$$=5-2\sqrt{6}$$

(viii)

$$\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}\times\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}}$$

$$=\frac{6+5-2\sqrt{30}}{\left(\sqrt{6}\right)^2-\left(\sqrt{5}\right)^2}=\frac{11-2\sqrt{30}}{6-5}=11-2\sqrt{30}$$

(ix)

$$\frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}-3\sqrt{2}}\times\frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}+3\sqrt{2}}=\frac{\left(2\sqrt{5}+3\sqrt{2}\right)^2}{\left(2\sqrt{5}\right)^2-\left(3\sqrt{2}\right)^2}$$

$$=\frac{4\times5+9\times2+12\sqrt{10}}{20-18}$$

$$=\frac{20+18+12\sqrt{10}}{2}=\frac{38+12\sqrt{10}}{2}=\frac{\cancel{2}\left(19+6\sqrt{10}\right)}{\cancel{2}}$$

$$=19+6\sqrt{10}$$

Question 4

Find the values of 'a' and 'b' in each of the following:

(i) $\frac{2+\sqrt{3}}{2-\sqrt{3}} = a + b\sqrt{3}$

(ii) $\frac{\sqrt{7}-2}{\sqrt{7}+2} = a\sqrt{7} + b$

(iii) $\frac{3}{\sqrt{3}-\sqrt{2}} = a\sqrt{3} - b\sqrt{2}$

(iv) $\frac{5+3\sqrt{2}}{5-3\sqrt{2}} = a + b\sqrt{2}$

Solution 4

(i) $\frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = a + b\sqrt{3}$

$\frac{(2+\sqrt{3})^2}{(2)^2 - (\sqrt{3})^2} = a + b\sqrt{3}$

$\frac{4+3+4\sqrt{3}}{4-3} = a + b\sqrt{3}$

$7 + 4\sqrt{3} = a + b\sqrt{3}$

$a = 7,\ b = 4$

(ii) $\frac{\sqrt{7}-2}{\sqrt{7}+2} \times \frac{\sqrt{7}-2}{\sqrt{7}-2} = a\sqrt{7} + b$

$\frac{(\sqrt{7}-2)^2}{(\sqrt{7})^2 - (2)^2} = a\sqrt{7} + b$

$\frac{7+4-4\sqrt{7}}{7-4} = a\sqrt{7} + b$

$\frac{11-4\sqrt{7}}{3} = a\sqrt{7} + b$

$a = \frac{-4}{3},\ b = \frac{11}{3}$

(iii) $\frac{3}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = a\sqrt{3} - b\sqrt{2}$

$$\frac{3\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}=a\sqrt{3}-b\sqrt{2}$$

$$\frac{3\left(\sqrt{3}+\sqrt{2}\right)}{3-2}=a\sqrt{3}-b\sqrt{2}$$

$$\left(3\sqrt{3}+3\sqrt{2}\right)=a\sqrt{3}-b\sqrt{2}$$

$$\Rightarrow a=3, b=-3$$

(iv) $$\frac{5+3\sqrt{2}}{5-3\sqrt{2}}\times\frac{5+3\sqrt{2}}{5+3\sqrt{2}}=a+b\sqrt{2}$$

$$\frac{\left(5+3\sqrt{2}\right)^2}{(5)^2-\left(3\sqrt{2}\right)^2}=a+b\sqrt{2}$$

$$\frac{25+18+30\sqrt{2}}{25-18}=a+b\sqrt{2}$$

$$\frac{43+30\sqrt{2}}{7}=a+b\sqrt{2}$$

$$a=\frac{43}{7},\quad b=\frac{30}{7}$$

Question 5

Simplify:

(i) $$\frac{22}{2\sqrt{3}+1}+\frac{17}{2\sqrt{3}-1}$$

(ii) $$\frac{\sqrt{2}}{\sqrt{6}-\sqrt{2}}-\frac{\sqrt{3}}{\sqrt{6}+\sqrt{2}}$$

Solution 5

(i)

$$\frac{22}{2\sqrt{3}+1}+\frac{17}{2\sqrt{3}-1}$$

$$\frac{22\left(2\sqrt{3}-1\right)+17\left(2\sqrt{3}+1\right)}{\left(2\sqrt{3}+1\right)\left(2\sqrt{3}-1\right)}=\frac{44\sqrt{3}-22+34\sqrt{3}+17}{\left(2\sqrt{3}\right)^2-1}$$

$$=\frac{78\sqrt{3}-5}{12-1}=\frac{78\sqrt{3}-5}{11}$$

(ii)

$$\frac{\sqrt{2}}{\sqrt{6}-2}-\frac{\sqrt{3}}{\sqrt{6}+\sqrt{2}}=\frac{\sqrt{2}\left(\sqrt{6}+\sqrt{2}\right)-\sqrt{3}\left(\sqrt{6}-\sqrt{2}\right)}{\left(\sqrt{6}\right)^2-\left(\sqrt{2}\right)^2}$$

$$= \frac{\sqrt{12}+2-\sqrt{18}+\sqrt{6}}{6-2} = \frac{2\sqrt{3}+2-3\sqrt{2}+\sqrt{6}}{4}$$

Question 6

If= $\frac{\sqrt{5}-2}{\sqrt{5}+2}$ and y = $\frac{\sqrt{5}+2}{\sqrt{5}-2}$; find:

(i) x^2(ii) y^2

(iii) xy(iv) $x^2 + y^2 + xy$.

Solution 6

(i) $x^2 = \left(\frac{\sqrt{5}-2}{\sqrt{5}+2}\right)^2 = \frac{5+4-4\sqrt{5}}{5+4+4\sqrt{5}} = \frac{9-4\sqrt{5}}{9+4\sqrt{5}}$

$$= \frac{9-4\sqrt{5}}{9+4\sqrt{5}} \times \left(\frac{9-4\sqrt{5}}{9-4\sqrt{5}}\right) = \frac{\left(9-4\sqrt{5}\right)^2}{(9)^2-\left(4\sqrt{5}\right)^2}$$

$$= \frac{81+80-72\sqrt{5}}{81-80} = 161-72\sqrt{5}$$

(ii) $y^2 = \left(\frac{\sqrt{5}+2}{\sqrt{5}-2}\right)^2 = \frac{5+4+4\sqrt{5}}{5+4-4\sqrt{5}} = \frac{9+4\sqrt{5}}{9-4\sqrt{5}}$

$$= \frac{9+4\sqrt{5}}{9-4\sqrt{5}} \times \frac{9+4\sqrt{5}}{9+4\sqrt{5}} = \frac{\left(9+4\sqrt{5}\right)^2}{(9)^2-\left(4\sqrt{5}\right)^2} = \frac{81+80+72\sqrt{5}}{81-80}$$

$$= 161+72\sqrt{5}$$

(iii) xy = $\frac{\left(\sqrt{5}-2\right)}{\left(\sqrt{5}+2\right)}\frac{\left(\sqrt{5}+2\right)}{\left(\sqrt{5}-2\right)} = 1$

(iv) $x^2 + y^2 + xy = 161 - 72\sqrt{5} + 161 + 72\sqrt{5} + 1$

= 322 + 1 = 323

Question 7

If m = $\frac{1}{3-2\sqrt{2}}$ and n = $\frac{1}{3+2\sqrt{2}}$, find:

(i) m^2

(ii) n^2

(iii) mn

Solution 7

$$\text{(i) } m = \frac{1}{3-2\sqrt{2}}$$

$$= \frac{1}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}}$$

$$= \frac{3+2\sqrt{2}}{(3)^2-(2\sqrt{2})^2}$$

$$= \frac{3+2\sqrt{2}}{9-8}$$

$$= 3+2\sqrt{2}$$

$$\Rightarrow m^2 = (3+2\sqrt{2})^2$$

$$= (3)^2 + 2\times 3\times 2\sqrt{2} + (2\sqrt{2})^2$$

$$= 9 + 12\sqrt{2} + 8$$

$$= 17 + 12\sqrt{2}$$

$$\text{(ii) } n = \frac{1}{3+2\sqrt{2}}$$

$$= \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$

$$= \frac{3-2\sqrt{2}}{(3)^2-(2\sqrt{2})^2}$$

$$= \frac{3+2\sqrt{2}}{9-8}$$

$$= 3-2\sqrt{2}$$

$$\Rightarrow n^2 = (3-2\sqrt{2})^2$$

$$= (3)^2 - 2\times 3\times 2\sqrt{2} + (2\sqrt{2})^2$$

$$= 9 - 12\sqrt{2} + 8$$

$$= 17 - 12\sqrt{2}$$

$$\text{(iii) } mn = (3+2\sqrt{2})(3-2\sqrt{2}) = (3)^2 - (2\sqrt{2})^2 = 9-8 = 1$$

Question 8

If $x = 2\sqrt{3} + 2\sqrt{2}$, find:

(i) $\frac{1}{x}$ (ii) $x + \frac{1}{x}$ (iii) $\left(x + \frac{1}{x}\right)^2$

Solution 8

(i) $\frac{1}{x} = \frac{1}{2\sqrt{3} + 2\sqrt{2}} \times \frac{2\sqrt{3} - \sqrt{2}}{2\sqrt{3} - 2\sqrt{2}} = \frac{2\sqrt{3} - 2\sqrt{2}}{12 - 8}$

$= \frac{\cancel{2}\left(\sqrt{3} - \sqrt{2}\right)}{\cancel{4}2} = \frac{\sqrt{3} - \sqrt{2}}{2}$

(ii) $x + \frac{1}{x} = 2\sqrt{3} + 2\sqrt{2} + \frac{\sqrt{3} - \sqrt{2}}{2}$

$= 2\left(\sqrt{3} + \sqrt{2}\right) + \frac{\left(\sqrt{3} - \sqrt{2}\right)}{2}$

$= \frac{4\left(\sqrt{3} + \sqrt{2}\right) + \left(\sqrt{3} - \sqrt{2}\right)}{2}$

$= \frac{4\sqrt{3} + 4\sqrt{2} + \sqrt{3} - \sqrt{2}}{2}$

$= \frac{5\sqrt{3} + 3\sqrt{2}}{2}$

(iii) $\left(x + \frac{1}{x}\right)^2 = \left(\frac{5\sqrt{3} + 3\sqrt{2}}{2}\right)^2 = \frac{75 + 18 + 30\sqrt{6}}{4}$

$= \frac{93 + 30\sqrt{6}}{4}$

Question 9

If $x = 1 - \sqrt{2}$, find the value of $\left(x - \frac{1}{x}\right)^3$

Solution 9

Given that $x = 1 - \sqrt{2}$

We need to find the value of $\left(x - \frac{1}{x}\right)^3$.

Since $x = 1 - \sqrt{2}$, we have

$$\frac{1}{x} = \frac{1}{1 - \sqrt{2}} \times \frac{1 + \sqrt{2}}{1 + \sqrt{2}}$$

$$\Rightarrow \frac{1}{x} = \frac{1 + \sqrt{2}}{1^2 - \left(\sqrt{2}\right)^2} \qquad \left[\text{Since } (a-b)(a+b) = a^2 - b^2\right]$$

$$\Rightarrow \frac{1}{x} = \frac{1 + \sqrt{2}}{1 - 2}$$

$$\Rightarrow \frac{1}{x} = \frac{1 + \sqrt{2}}{-1}$$

$$\Rightarrow \frac{1}{x} = -\left(1 + \sqrt{2}\right) \ldots (1)$$

Thus, $\left(x - \frac{1}{x}\right) = \left(1 - \sqrt{2}\right) - \left(-\left(1 + \sqrt{2}\right)\right)$

$$\Rightarrow \left(x - \frac{1}{x}\right) = 1 - \sqrt{2} + 1 + \sqrt{2}$$

$$\Rightarrow \left(x - \frac{1}{x}\right) = 2$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^3 = 2^3$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^3 = 8$$

Question 10

If $x = 5 - 2\sqrt{6}$, find $x^2 + \frac{1}{x^2}$.

Solution 10

Given $x = 5 - 2\sqrt{6}$

We need to find $x^2 + \frac{1}{x^2}$:

Since $x = 5 - 2\sqrt{6}$, we have

$$\frac{1}{x} = \frac{1}{5 - 2\sqrt{6}}$$

$$\Rightarrow \frac{1}{x} = \frac{1}{5 - 2\sqrt{6}} \times \frac{5 + 2\sqrt{6}}{5 + 2\sqrt{6}}$$

$$\Rightarrow \frac{1}{x} = \frac{5 + 2\sqrt{6}}{(5 - 2\sqrt{6})(5 + 2\sqrt{6})}$$

$$\Rightarrow \frac{1}{x} = \frac{5 + 2\sqrt{6}}{5^2 - (2\sqrt{6})^2}$$

$$\Rightarrow \frac{1}{x} = \frac{5 + 2\sqrt{6}}{25 - 24}$$

$$\Rightarrow \frac{1}{x} = 5 + 2\sqrt{6} \ldots (1)$$

Thus, $\left(x - \frac{1}{x}\right) = (5 - 2\sqrt{6}) - (5 + 2\sqrt{6})$

$$\Rightarrow \left(x - \frac{1}{x}\right) = 5 - 2\sqrt{6} - 5 - 2\sqrt{6}$$

$$\Rightarrow \left(x - \frac{1}{x}\right) = -4\sqrt{6} \ldots (2)$$

Now consider $\left(x - \frac{1}{x}\right)^2$:

Thus

$$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2x \times \frac{1}{x} \quad \left[\text{since } (a - b)^2 = a^2 - 2ab + b^2\right]$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 + 2 = x^2 + \frac{1}{x^2} \ldots (3)$$

Thus, from equations (2) and (3), we have

$$x^2 + \frac{1}{x^2} = (-4\sqrt{6})^2 + 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 96 + 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 98$$

Question 11

Show that:

$$\frac{1}{3-2\sqrt{2}}-\frac{1}{2\sqrt{2}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}=5.$$

Solution 11

$$\text{L.H.S.}=\frac{1}{3-2\sqrt{2}}-\frac{1}{2\sqrt{2}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$$

$$=\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$$

$$=\frac{1}{3-\sqrt{8}}\times\frac{3+\sqrt{8}}{3+\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}\times\frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}\times\frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}$$

$$-\frac{1}{\sqrt{6}-\sqrt{5}}\times\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}+\frac{1}{\sqrt{5}-2}\times\frac{\sqrt{5}+2}{\sqrt{5}+2}$$

$$=\frac{3+\sqrt{8}}{(3)^2-(\sqrt{8})^2}-\frac{\sqrt{8}+\sqrt{7}}{(\sqrt{8})^2-(\sqrt{7})^2}+\frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2}-\frac{\sqrt{6}+\sqrt{5}}{(\sqrt{6})^2-(\sqrt{5})^2}+\frac{\sqrt{5}+2}{(\sqrt{5})^2-(2)^2}$$

$$=\frac{3+\sqrt{8}}{9-8}-\frac{\sqrt{8}+\sqrt{7}}{8-7}+\frac{\sqrt{7}+\sqrt{6}}{7-6}-\frac{\sqrt{6}+\sqrt{5}}{6-5}+\frac{\sqrt{5}+2}{5-4}$$

$$=3+\sqrt{8}-\sqrt{8}-\sqrt{7}+\sqrt{7}+\sqrt{6}-\sqrt{6}-\sqrt{5}+\sqrt{5}+2$$

$$=3+2$$

$$=5$$

$$=\text{R.H.S.}$$

Question 12

Rationalise the denominator of: $\frac{1}{\sqrt{3}-\sqrt{2}+1}$

Solution 12

$$\frac{1}{\sqrt{3}-\sqrt{2}+1}$$

$$= \frac{1}{\left(\sqrt{3}-\sqrt{2}\right)+1} \times \frac{\left(\sqrt{3}-\sqrt{2}\right)-1}{\left(\sqrt{3}-\sqrt{2}\right)-1}$$

$$= \frac{\sqrt{3}-\sqrt{2}-1}{\left(\sqrt{3}-\sqrt{2}\right)^2-(1)^2}$$

$$= \frac{\sqrt{3}-\sqrt{2}-1}{\left(\sqrt{3}\right)^2-2\sqrt{6}+\left(\sqrt{2}\right)^2-1}$$

$$= \frac{\sqrt{3}-\sqrt{2}-1}{3-2\sqrt{6}+2-1}$$

$$= \frac{\sqrt{3}-\sqrt{2}-1}{4-2\sqrt{6}}$$

$$= \frac{\left(\sqrt{3}-\sqrt{2}\right)-1}{2\left(2-\sqrt{6}\right)}$$

$$= \frac{\sqrt{3}-\sqrt{2}-1}{2\left(2-\sqrt{6}\right)} \times \frac{2+\sqrt{6}}{2+\sqrt{6}}$$

$$= \frac{2\sqrt{3}-2\sqrt{2}-2+\sqrt{18}-\sqrt{12}-\sqrt{6}}{2\left[(2)^2-\left(\sqrt{6}\right)^2\right]}$$

$$= \frac{2\sqrt{3}-2\sqrt{2}-2+3\sqrt{2}-2\sqrt{3}-\sqrt{6}}{2(4-6)}$$

$$= \frac{\sqrt{2}-2-\sqrt{6}}{2(-2)}$$

$$= \frac{\sqrt{2}-2-\sqrt{6}}{-4}$$

$$= \frac{1}{4}\left(2+\sqrt{6}-\sqrt{2}\right)$$

Question 13

If $\sqrt{2} = 1.4$ and $\sqrt{3 = 1.7}$,find the value of each of the following ,correct to one decimal place:

(i) $\frac{1}{\sqrt{3}-\sqrt{2}}$

(ii) $\frac{1}{3+2\sqrt{2}}$

(iii) $\frac{2-\sqrt{3}}{\sqrt{3}}$

Solution 13

(i)

$\sqrt{2} = 1.4$ and $\sqrt{3} = 1.7$

$$\frac{1}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{1}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$= \frac{\sqrt{3}+\sqrt{2}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}$$

$$= \frac{\sqrt{3}+\sqrt{2}}{3-2}$$

$$= \sqrt{3}+\sqrt{2}$$

$$= 1.7 + 1.4$$

$$= 3.1$$

$\sqrt{2} = 1.4$ and $\sqrt{3} = 1.7$

(ii) $\frac{1}{3+2\sqrt{2}}$

$$= \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$

$$= \frac{3-2\sqrt{2}}{(3)^2-\left(2\sqrt{2}\right)^2}$$

$$= \frac{3-2\sqrt{2}}{9-8}$$

$$= 3-2\sqrt{2}$$

$$= 3-2(1.4)$$

$$= 3-2.8$$

$$= 0.2$$

(iii)

$$\frac{2-\sqrt{3}}{\sqrt{3}}$$

$$= \frac{2-\sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{(2 - 1.7)1.7}{3}$$

$$= \frac{0.3 \times 1.7}{3}$$

$$= 0.17$$

$$= 0.2$$

Question 14

Evaluate :

$$\frac{4-\sqrt{5}}{4+\sqrt{5}} + \frac{4+\sqrt{5}}{4-\sqrt{5}}.$$

Solution 14

$$\frac{4-\sqrt{5}}{4+\sqrt{5}} + \frac{4+\sqrt{5}}{4-\sqrt{5}}$$

$$= \frac{4-\sqrt{5}}{4+\sqrt{5}} \times \frac{4-\sqrt{5}}{4-\sqrt{5}} + \frac{4+\sqrt{5}}{4-\sqrt{5}} \times \frac{4+\sqrt{5}}{4+\sqrt{5}}$$

$$= \frac{\left(4-\sqrt{5}\right)^2}{4^2-\left(\sqrt{5}\right)^2} + \frac{\left(4+\sqrt{5}\right)^2}{4^2-\left(\sqrt{5}\right)^2}$$

$$= \frac{16+5-8\sqrt{5}}{16-5} + \frac{16+5+8\sqrt{5}}{16-5}$$

$$= \frac{21-8\sqrt{5}}{11} + \frac{21+8\sqrt{5}}{11}$$

$$= \frac{21-8\sqrt{5}+21+8\sqrt{5}}{11}$$

$$= \frac{42}{11}$$

$$= 3\frac{9}{11}$$

Question 15

If $\frac{2+\sqrt{5}}{2-\sqrt{5}} = x$ and $\frac{2-\sqrt{5}}{2+\sqrt{5}} = y$; find the value of $x^2 - y^2$.

Solution 15

$$x = \frac{2+\sqrt{5}}{2-\sqrt{5}} \qquad y = \frac{2-\sqrt{5}}{2+\sqrt{5}}$$

$$= \frac{2+\sqrt{5}}{2-\sqrt{5}} \times \frac{2+\sqrt{5}}{2+\sqrt{5}} \qquad = \frac{2-\sqrt{5}}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$$

$$= \frac{(2+\sqrt{5})^2}{2^2-(\sqrt{5})^2} \qquad = \frac{(2-\sqrt{5})^2}{2^2-(\sqrt{5})^2}$$

$$= \frac{4+4\sqrt{5}+5}{4-5} \qquad = \frac{4-4\sqrt{5}+5}{4-5}$$

$$= \frac{9+4\sqrt{5}}{-1} \qquad = \frac{9-4\sqrt{5}}{-1}$$

$$= -9-4\sqrt{5} \qquad = -9+4\sqrt{5}$$

$$\therefore x^2 - y^2 = (-9-4\sqrt{5})^2 - (-9+4\sqrt{5})^2$$

$$= 81+72\sqrt{5}+80-(81-72\sqrt{5}+80)$$

$$= 81+72\sqrt{5}+80-81+72\sqrt{5}-80$$

$$= 144\sqrt{5}$$

Chapter 2 - Compound Interest (Without using formula)

Exercise Ex. 2(A)

Question 1

Rs.16,000 is invested at 5% compound interest compounded per annum.
Use the table, given below, to find the amount in 4 years.

Year ↓	Initial amount (Rs.)	Interest (Rs.)	Final amount (Rs.)
1 st 2 nd 3 rd 4 th 5 th	16,000	800	16,800

Solution 1

Year ↓	Initial amount (Rs.)	Interest (Rs.)	Final amount (Rs.)
1 st	16,000	800	16,800
2 nd	16,800	840	17,640
3 rd	17,640	882	18,522
4 th	18,522	926.10	**19448.10**
5 th	19448.10	972.405	20420.505

Thus, the amount in 4 years is Rs. 19448.10.

Question 2(i)

Calculate the amount and the compound interest on :

Rs. 6,000 in 3 years at 5% per year.

Solution 2(i)

For 1st year,

$P = \text{Rs. } 6,000; \quad R = 5\% \text{ and } T = 1 \text{ year}$

$\therefore \text{Interest} = \text{Rs. } \frac{6,000 \times 5 \times 1}{100} = \text{Rs. } 300$

And, amount $= \text{Rs. } (6,000 + 300) = \text{Rs. } 6,300$

For 2nd year,

$P = \text{Rs. } 6,300; \quad R = 5\% \text{ and } T = 1 \text{ year}$

$\therefore \text{Interest} = \text{Rs. } \frac{6,300 \times 5 \times 1}{100} = \text{Rs. } 315$

And, amount $= \text{Rs. } (6,300 + 315) = \text{Rs. } 6,615$

For 3rd year,

$P = \text{Rs. } 6,615; \quad R = 5\% \text{ and } T = 1 \text{ year}$

$\therefore \text{Interest} = \text{Rs. } \frac{6,615 \times 5 \times 1}{100} = \text{Rs. } 330.75$

And, amount $= \text{Rs. } (6,615 + 330.75) = \text{Rs. } 6,945.75$

$\therefore$ C.I. accrued = Final amount – Intitial Principal

$= \text{Rs. } (6,945.75 - 6,000)$

$= \text{Rs. } 945.75$

Question 2(ii)

Calculate the amount and the compound interest on :

Rs. 8,000 in $2\frac{1}{2}$ years at 15% per annum.

Solution 2(ii)

For 1st year,

$P = Rs.\ 8,000;\quad R = 15\%\ \text{and}\ T = 1\ \text{year}$

$\therefore\ \text{Interest} = Rs.\ \frac{8,000 \times 15 \times 1}{100} = Rs.\ 1200$

And, amount $= Rs.\ (8,000 + 1200) = Rs.\ 9,200$

For 2nd year,

$P = Rs.\ 9,200;\quad R = 15\%\ \text{and}\ T = 1\ \text{year}$

$\therefore\ \text{Interest} = Rs.\ \frac{9,200 \times 15 \times 1}{100} = Rs.\ 1,380$

And, amount $= Rs.\ (9,200 + 1,380) = Rs.\ 10,580$

For the last $\frac{1}{2}$ year,

$P = Rs.\ 10,580;\quad R = 15\%\ \text{and}\ T = \frac{1}{2}\ \text{year}$

$\therefore\ \text{Interest} = Rs.\ \frac{10,580 \times 15 \times 1}{100 \times 2} = Rs.\ 793.50$

And, amount $= Rs.\ (10,580 + 793.50) = Rs.\ 11373.50$

$\therefore$ C.I. accrued = Final amount − Intitial Principal

$= Rs.\ (11373.50 - 8,000)$

$= Rs.\ 3373.50$

Question 3

Calculate the amount and the compound interest on:
(i) ₹ 4,600 in 2 years when the rates of interest of successive years are 10%and 12% respectively.
(ii) ₹ 16,000 in 3 years, when the rates of the interest for successive years are 10%, 14% and 15% respectively.

Solution 3

(i)
For 1st year
P = Rs. 4600
R = 10%
T = 1 year.

$$I = \frac{4600 \times 10 \times 1}{100} = Rs.460$$

A = 4600 + 460 = Rs. 5060
For 2nd year
P = Rs. 5060
R = 12%
T = 1 year.

$$I = \frac{5060 \times 12 \times 1}{100} = \frac{60720}{100} = 607.20$$

A= 5060 + 607.20 = Rs. 5667.20
Compound interest = 5667.20 - 4600
= Rs. 1067.20
Amount after 2 years = Rs. 5667.20
(ii)
For 1st year
P = Rs. 16000
R = 10%
T = 1 year

$$I = \frac{16000 \times 10 \times 1}{100} = Rs.1600$$

A = 16000 + 1600 = 17600
For 2nd year,
P = Rs. 17600
R = 14%
T = 1 year

$$I = \frac{17600 \times 14 \times 1}{100} = \frac{246400}{100} = Rs.2464.$$

A = 1760 + 24654 = Rs. 20064
For 3rd year,
P = Rs. 20064
R = 15%
T = 1 year

$$I = \frac{20064 \times 15 \times 1}{100} = 3009.60$$

Amount after 3 years = 20064 + 3009.60
= Rs. 23073.60
Compound interest = 23073.60 - 16000
= Rs. 7073.60

Question 4

Find the compound interest, correct to the nearest rupee, on ₹ 2,400 for $2\frac{1}{2}$ years at 5 per cent per annum.

Solution 4

For 1st years
P = Rs. 2400
R = 5%
T = 1 year

$$I = \frac{2400 \times 5 \times 1}{100} = 120$$

A = 2400 + 120 = Rs. 2520
For 2nd year
P = Rs. 2520
R = 5%
T = 1 year

$$I = \frac{2520 \times 5 \times 1}{100} = Rs.126$$

A = 2520 + 126 = Rs. 2646
For final $\frac{1}{2}$ year,
P = Rs. 2646
R = 5%
T = $\frac{1}{2}$ year

$$I = \frac{2646 \times 5 \times 1}{100 \times 2} = Rs.66.15$$

Amount after $2\frac{1}{2}$ years = 2646 + 66.15
= Rs. 2712.15
Compound interest = 2712.15 - 2400
= Rs. 312.15

Question 5

Calculate the compound interest for the second year on ₹ 8,000/- invested for 3 years at 10% per annum.

Solution 5

For 1st year
P = Rs. 8000
R = 10%
T = 1 year

$$I = \frac{8000 \times 10 \times 1}{100} = 800$$

A = 8000 + 800 = Rs. 8800
For 2nd year
P = Rs. 8800
R = 10%
T = 1 year

$$I = \frac{8800 \times 10 \times 1}{100}$$

Compound interest for 2nd years = Rs. 880

Question 6

A borrowed ₹2,500 from B at 12% per annum compound interest. After 2 years, A gave ₹2,936 and a watch to B to clear the account. Find the cost of the watch.

Solution 6

For 1st year
P = Rs. 2500
R = 12%
T = 1 year

$$I = \frac{2500 \times 12 \times 1}{100} = Rs.300$$

Amount = 2500 + 300 = Rs. 2800
For 2nd year
P = Rs. 2800
R = 12%
T = 1 year

$$I = \frac{2800 \times 12 \times 1}{100} = Rs.336$$

Amount = 2800 + 336 = Rs. 3136
Amount repaid by A to B = Rs. 2936
The amount of watch =Rs. 3136 - Rs. 2936 = Rs. 200

Question 7

How much will Rs. 50,000 amount to in 3 years, compounded yearly, if the rates for the successive years are 6%, 8% and 10% respectively?

Solution 7

$$\text{Interest for the first year} = \frac{P \times R \times T}{100}$$
$$= \frac{50,000 \times 6 \times 1}{100}$$
$$= \text{Rs. } 3,000$$

Amount for the first year = Rs. 50,000 + Rs. 3,000 = Rs. 53,000

$$\text{Interest for the second year} = \frac{P \times R \times T}{100}$$
$$= \frac{53,000 \times 8 \times 1}{100}$$
$$= \text{Rs. } 4,240$$

Amount for the second year = Rs. 53,000 + Rs. 4,240 = Rs. 57,240

$$\text{Interest for the third year} = \frac{P \times R \times T}{100}$$
$$= \frac{57,240 \times 10 \times 1}{100}$$
$$= \text{Rs. } 5,724$$

Amount for the third year = Rs. 57,240 + Rs. 5,724 = Rs. 62,964

Hence, the amount will be Rs. 62,964.

Question 8

Meenal lends Rs. 75,000 at C.I. for 3 years. If the rate of interest for the first two years is 15% per year and for the third year it is 16%, calculate the sum Meenal will get at the end of the third year.

Solution 8

$$\text{Interest for the first year} = \frac{P \times R \times T}{100}$$
$$= \frac{75,000 \times 15 \times 1}{100}$$
$$= \text{Rs. } 11,250$$

Amount for the first year = Rs. 75,000 + Rs. 3,000 = Rs. 86,250

$$\text{Interest for the second year} = \frac{P \times R \times T}{100}$$
$$= \frac{86,250 \times 15 \times 1}{100}$$
$$= \text{Rs. } 12,937.5$$

Amount for the second year = Rs. 86,250 + Rs. 12,937.5 = Rs. 99,187.5

$$\text{Interest for the third year} = \frac{P \times R \times T}{100}$$
$$= \frac{99,187.5 \times 16 \times 1}{100}$$
$$= \text{Rs. } 15,870$$

Amount for the third year = Rs. 99,187.5 + Rs. 15,870 = Rs. 1,15,057.5

Hence, the sum Meenal will get at the end of the third year is Rs. 1,15,057.5.

Question 9

Govind borrows Rs18,000 at 10% simple interest. He immediately invests the money borrowed at 10% compound interest compounded half-yearly. How much money does Govind gain in one year ?

Solution 9

To calculate S.I.

P=Rs18,000; R=10% and T=1year

$$\text{S.I.} = \text{Rs}\ \frac{18,000 \times 10 \times 1}{100} = \text{Rs}1,800$$

To calculate C.I.

For 1st half- year

P= Rs18,000; R=10% and T= 1/2year

$$\text{Interest} = \text{Rs}\ \frac{18,000 \times 10 \times 1}{100 \times 2} = \text{Rs}900$$

Amount= Rs18,000+ Rs900= Rs18,900

For 2nd year

P= Rs18,900; R= 10% and T= 1/2year

$$\text{Interest} = \text{Rs}\ \frac{18,900 \times 10 \times 1}{100 \times 2} = \text{Rs}945$$

Amount= Rs18,900+ Rs945= Rs19,845

∴ Compound interest= Rs19,845- Rs18,000= Rs1,845

∴ His gain= Rs1,845 - Rs1,800= Rs45

Question 10

Find the compound interest on Rs. 4,000 accrued in three years, when the rate of interest is 8% for the first year and 10% per year for the second and the third years.

Solution 10

$$\text{Interest for the first year} = \frac{P \times R \times T}{100}$$
$$= \frac{4,000 \times 8 \times 1}{100}$$
$$= \text{Rs. } 320$$

$$\text{Amount for the first year} = \text{Rs. } 4,000 + \text{Rs. } 320 = \text{Rs. } 4,320$$

$$\text{Interest for the second year} = \frac{P \times R \times T}{100}$$
$$= \frac{4,320 \times 10 \times 1}{100}$$
$$= \text{Rs. } 432$$

$$\text{Amount for the second year} = \text{Rs. } 4,320 + \text{Rs. } 432 = \text{Rs. } 4,752$$

$$\text{Interest for the third year} = \frac{P \times R \times T}{100}$$
$$= \frac{4,752 \times 10 \times 1}{100}$$
$$= \text{Rs. } 475.20$$

$$\text{Amount for the third year} = \text{Rs. } 4,752 + \text{Rs. } 475.20 = \text{Rs. } 5,227.20$$

$$\text{So, the compound interest} = \text{Rs. } 5,227.20 - \text{Rs. } 4,000 = \text{Rs. } 1,227.20$$

Hence, the sum Meenal will get at the end of the third year is Rs. 1,227.20.

Chapter 2 - Compound Interest (Without using formula)

Exercise Ex. 2(B)

Question 1

Calculate the difference between the simple interest and the compound interest on ₹ 4,000 in 2 years at 8% per annum compounded yearly.

Solution 1

For 1st year

P = Rs. 4000

R = 8

T = 1 year

$$I = \frac{4000 \times 8 \times 1}{100} = 320$$

A = 4000 + 320 = Rs. 4320
For 2nd year
P = Rs. 4320
R=8%
T = 1 year

$$I = \frac{4320 \times 8 \times 1}{100} = Rs.345.60$$

A = 4320 + 345.60 = 4665.60
Compound interest = Rs. 4665.60 - Rs. 4000
= Rs. 665.60

Simple interest for 2 years = $\frac{4000 \times 8 \times 2}{100}$

= Rs. 640
Difference of CI and SI = 665.60 - 640
= Rs 25.60

Question 2

A man lends ₹ 12,500 at 12% for the first year, at 15% for the second year and at 18% for the third year. If the rates of interest are compounded yearly ; find the difference between the C.I. fo the first year and the compound interest for the third year.

Solution 2

For 1st year
P = Rs. 12500
R = 12%
R = 1 year

$$I = \frac{12500 \times 12 \times 1}{100} = Rs.1500$$

A = 12500 + 1500 = Rs. 14000
For 2nd year
P = Rs. 1400
R = 15%
T = 1 year

$$I = \frac{14000 \times 15 \times 1}{100} = Rs.2898$$

A = 1400 + 2100 = Rs. 16100

For 3rd year
P = Rs. 16100
R = 18%
T = 1 year

$$I = \frac{16100 \times 18 \times 1}{100} = Rs.2898$$

A = 16100 + 2898 = Rs. 3998
Difference between the compound interest of the third year and first year
= Rs. 2893 - Rs. 1500
= Rs. 1398

Question 3

A sum of money is lent at 8% per annum compound interest. If the interest for the second year exceeds that for the first year by Rs.96, find the sum of money.

Solution 3

Let money be Rs100
For 1st year
P=Rs100; R=8% and T= 1year

Interest for the first year= Rs $\frac{100 \times 8 \times 1}{100}$ = Rs8
Amount= Rs100+ Rs8= Rs108
For 2nd year
P=Rs108; R=8% and T= 1year

Interest for the second year= Rs $\frac{108 \times 8 \times 1}{100}$ = Rs8.64
Difference between the interests for the second and first year = Rs8.64 - Rs8 = Rs0.64
Given that interest for the second year exceeds the first year by Rs.96
When the difference between the interests is Rs0.64, principal is Rs100

When the difference between the interests is Rs96, principal=Rs $\frac{96 \times 100}{0.64}$ =Rs15,000

Question 4

A man borrows Rs. 6,000 at 5% C.I. per annum. If he repays Rs. 1,200 at the end of each year, find the amount of the loan outstanding at the beginning of the third year.

Solution 4

Given that the amount borrowed = Rs. 6, 000

Rate per annum = 5%

Interest on Rs. 6, 000 = $\frac{5}{100}$ x Rs. 6, 000 = Rs. 300

So, amount at the end of the first year

= Rs. 6, 000 + Rs. 300

= Rs. 6, 300

Amount left to be paid = Rs. 6, 300 - Rs. 1, 200

= Rs. 5, 100

Interest on Rs. 5, 100 = $\frac{5}{100}$ x Rs. 5, 100 = Rs. 255

So, amount at the end of the second year

= Rs. 5, 100 + Rs. 255

= Rs. 5, 355

Amount left to be paid = Rs. 5, 355 - Rs. 1, 200

= Rs. 4, 155

Hence, the amount of the loan outstanding at the beginning of the third year is Rs. 4, 155.

Question 5

A man borrows Rs. 5,000 at 12 percent compound interest payable every six months. He repays Rs. 1,800 at the end of every six months. Calculate the third payment he has to make at the end of 18 months in order to clear the entire loan.

Solution 5

For 1st six months:

P = Rs. 5,000, R = 12% and T = year

∴ Interest = $\frac{5,000 \times 12 \times 1}{2 \times 100}$ = Rs. 300

And, Amount = Rs. 5,000 + Rs. 300 = Rs. 5,300

Since money repaid = Rs. 1,800

Balance = Rs. 5,300 - Rs. 1,800 = Rs. 3,500

For 2nd six months:

P = Rs. 3,500, R = 12% and T = year

∴ Interest = $\frac{3,500 \times 12 \times 1}{2 \times 100}$ = Rs. 210

And, Amount = Rs. 3,500 + Rs. 210 = Rs. 3,710

Again money repaid = Rs. 1,800

Balance = Rs. 3,710 - Rs. 1,800 = Rs. 1,910

For 3rd six months:

P = Rs. 1,910, R = 12% and T = year

$$\therefore \text{Interest} = \frac{1,910 \times 12 \times 1}{2 \times 100} = \text{Rs. } 114.60$$

And, Amount = Rs. 1,910 + Rs. 114.60 = Rs. 2,024.60

Thus, the 3rd payment to be made to clear the entire loan is 2,024.60.

Question 6

On a certain sum of money, the difference between the compound interest for a year, payable half-yearly, and the simple interest for a year is ₹ 180/-. Find the sum lent out, if the rate of interest in both the cases is 10% per annum.

Solution 6

Let principal (p = Rs. 100
R = 10%
T = 1 year

$$\text{SI} = \frac{100 \times 10 \times 1}{100} = \text{Rs.}10$$

Compound interest payable half yearly
R = 5% half yearly

$$\text{T} = \frac{1}{2} \text{ year} = 1 \text{ half year}$$

For first $\frac{1}{2}$ year

$$\text{I} = \frac{100 \times 5 \times 1}{100} = \text{Rs.}5$$

A = 100 + 5 = Rs. 105

For second $\frac{1}{2}$ year

P = Rs. 105

$$\text{I} = \frac{105 \times 5 \times 1}{100} = \text{Rs.}5.25$$

Total compound interest = 5 + 5.25
= Rs. 10.25
Difference of CI and SI = 10.25- 10
= Rs. 0.25
When difference in interest is Rs. 10.25, sum = Rs. 100

If the difference is Rs. 1 ,sum = $\frac{100}{0.25}$

If the difference is Rs. = 180,sum = $\frac{100}{0.25} \times 180$
= Rs. 72000

Question 7

A manufacturer estimates that his machine depreciates by 15% of its value at the beginning of the year. Find the original value (cost) of the machine, if it depreciates by Rs. 5,355 during the second year.

Solution 7

Let the original cost of the machine = Rs. 100

$\therefore$ Depreciation during the 1st year = 15% of Rs. 100 = Rs. 15

Value of the machine at the beginning of the 2nd year

= Rs. 100 – Rs. 15

= Rs. 85

$\therefore$ Depreciation during the 2nd year = 15% of Rs. 85 = Rs. 12.75

Now, when depreciation during 2nd year = Rs. 12.75, original cost = Rs. 100

$\Rightarrow$ when depreciation during 2nd year = Rs. 5, 355

original cost = Rs. $\frac{100}{12.75} \times 5,355$ = Rs. 42, 000

Hence, original cost of the machine is Rs. 42, 000.

Question 8

A man invest ₹ 5,600 at 14% per annum compound interest for 2 years. Calculate:
(i) The interest for the first year.
(ii) The amount at the end of the first year.
(iii) The interest for the second year, correct to the nearest rupee.

Solution 8

(i) For 1st years
P = Rs. 5600
R = 14%
T = 1 year

$$I = \frac{5600 \times 14 \times 1}{100} = Rs.784$$

(ii) Amount at the end of the first year
= 5600 + 784
= Rs. 6384
(iii) For 2nd year
P = 6384
R = 14%
R = 1 year

$$I = \frac{6384 \times 14 \times 1}{100}$$

= Rs. 803.76
= Rs. 894 (nearly)

Question 9

A man saves ₹3,000 every year and invests it at the end of the year at 10% compound interest. Calculate the total amount of his savings at the end of the third years.

Solution 9

Savings at the end of every year = Rs. 3000
For 2nd year
P = Rs. 3000
R = 10%
T = 1 year

$$I = \frac{3000 \times 10 \times 1}{100} = 300$$

A = 3000 + 300 = Rs. 3300
For third year, savings = 3000
P = 3000 + 3300 = Rs. 6300
R = 10%
T = 1 year

$$I = \frac{6300 \times 10 \times 1}{100} = Rs.630$$

A = 6300 + 630 = Rs. 6930
Amount at the end of 3rd year
= 6930 + 3000
= Rs. 9930

Question 10

A man borrows Rs. 10,000 at 5% per annum compound interest. He repays 35% of the sum borrowed at the end of the first year and 42% of the sum borrowed at the end of the second year. How much must he pay at the end of the third year in order to clear the debt?

Solution 10

The amount borrowed = Rs. 10, 000

Interest for the first year $= \frac{P \times R \times T}{100}$

$= \frac{10,000 \times 5 \times 1}{100}$

= Rs. 500

So, amount at the end of the first year

= Rs. 10, 000 + Rs. 500

= Rs. 10, 500

The man pays 35% of Rs. 10, 500 at the end of the first year

$= \frac{35}{100} \times 10,500$ = Rs. 3, 675

So, amount left to be paid

= Rs. 10, 500 – Rs. 3, 675 = Rs. 6, 825

Interest for the second year $= \frac{P \times R \times T}{100}$

$= \frac{6,825 \times 5 \times 1}{100}$

= Rs. 341.25

So, amount at the end of the second year

= Rs. 6, 825 + Rs. 341.25

= Rs. 7, 166.25

The man pays 42% of Rs. 7166.25 at the end of the second year

$= \frac{42}{100} \times 7166.25$ = Rs. 3,009.825

So, amount left to be paid

= Rs. 7, 166.25 – Rs. 3, 009.825 = Rs. 4, 156.425

Interest for the third year $= \frac{P \times R \times T}{100}$

$= \frac{4,156.425 \times 5 \times 1}{100}$

= Rs. 207. 82125

So, amount at the end of the third year

= Rs. 4, 156.425 + Rs. 207.82125

= Rs. 4, 364.24625

Hence, he must pay Rs. 4, 364.24625 at the end of the third year in order to clear the debt.

Chapter 2 - Compound Interest (Without using formula)

Exercise Ex. 2(C)

Question 1

A sum is invested at compound interest, compounded yearly. If the interest for two successive years is Rs.5,700 and Rs.7,410, calculate the rate of interest.

Solution 1

$$\text{Rate of interest} = \frac{\text{Difference in the interest of the two consecutive periods} \times 100}{\text{C.I. of preceeding year} \times \text{Time}}\%$$

$$= \frac{(7410 - 5700) \times 100}{5700 \times 1}\%$$

$$= 30\%$$

Question 2

A certain sum of money is put at compound interest, compounded half-yearly. If the interest for two successive half-years are Rs650 and Rs760.50; find the rate of interest.

Solution 2

∵ Difference between the C.I. of two successive half-years
= Rs760.50 - Rs650= Rs110.50
⇒Rs110.50 is the interest of one half-year on Rs650

∴ Rate of interest= Rs $\frac{100 \times I}{P \times T}$ %= $\frac{100 \times 110.50}{650 \times \frac{1}{2}}$ %= 34%

Question 3

A certain sum amounts to Rs5,292 in two years and Rs5,556.60 in three years, interest being compounded annually. Find;
(i)the rate of interest.
(ii)the original sum.

Solution 3

(i)Amount in two years= Rs5,292
Amount in three years= Rs5,556.60
∵ Difference between the amounts of two successive years
= Rs5,556.60 - Rs5,292= Rs264.60
⇒Rs264.60 is the interest of one year on Rs5,292

∴ Rate of interest= Rs $\frac{100 \times I}{P \times T}$ %= $\frac{100 \times 264.60}{5,292 \times 1}$ %= 5%

(ii) Let the sum of money= Rs100
∴ Interest on it for 1st year= 5% of Rs100= Rs5
⇒Amount in one year= Rs100+ Rs5= Rs105
Similarly, amount in two years= Rs105+ 5% of Rs105
= Rs105+ Rs5.25
= Rs110.25

When amount in two years is Rs110.25, sum = Rs100

$\Rightarrow$When amount in two years is Rs5,292, sum = Rs $\frac{100 \times 5,292}{110.25}$
= Rs4,800

Question 4

The compound interest, calculated yearly, on a certain sum of money for the second year is Rs1,089 and for the third year it is Rs1,197.90. Calculate the rate of interest and the sum of money.

Solution 4

(i)C.I. for second year = Rs1,089
C.I. for third year = Rs 1,197.90
$\because$ Difference between the C.I. of two successive years
= Rs1,197.90 - Rs1089= Rs108.90
$\Rightarrow$Rs108.90 is the interest of one year on Rs1089
$\therefore$ Rate of interest= Rs $\frac{100 \times I}{P \times T}$ %= $\frac{100 \times 108.90}{1089 \times 1}$ %= 10%
(ii) Let the sum of money= Rs100
$\therefore$ Interest on it for 1st year= 10% of Rs100= Rs10
$\Rightarrow$Amount in one year= Rs100+ Rs10= Rs110
Similarly, C.I. for 2nd year= 10% of Rs110
= Rs11
When C.I. for 2nd year is Rs11, sum = Rs100

$\Rightarrow$When C.I. for 2nd year is Rs1089, sum = Rs $\frac{100 \times 1089}{11}$ = Rs9,900

Question 5

Mohit invests Rs8,000 for 3years at a certain rate of interest, compounded annually. At the end of one year it amounts to Rs9,440. Calculate:
(i)the rate of interest per annum.
(ii)the amount at the end of the second year.
(iii)the interest accrued in the third year.

Solution 5

<u>For 1st year</u>
P=Rs8,000; A=9,440 and T= 1year
Interest= Rs9,440 - Rs8,000= Rs1,440
Rate= $\frac{I \times 100}{P \times T}$ %= $\frac{1,440 \times 100}{8,000 \times 1}$ %=18%
<u>For 2nd year</u>
P= Rs9,440; R=18% and T= 1year
Interest= Rs $\frac{9,440 \times 18 \times 1}{100}$ = Rs1,699.20
Amount= Rs9,440 + Rs1,699.20= Rs11,139.20
<u>For 3rd year</u>
P= Rs11,139.20; R=18% and T= 1year

Interest= Rs $\frac{11,139.20 \times 18 \times 1}{100}$ = Rs2,005.06

Question 6

Geeta borrowed Rs15,000 for 18 months at a certain rate of interest compounded semi-annually. If at the end of six months it amounted to Rs15,600; calculate :
(i)the rate of interest per annum.
(ii)the total amount of money that Geeta must pay at the end of 18 months in order to clear the account.

Solution 6

For 1st half-year
P= Rs15,000; A= Rs15,600 and T= ½ year
Interest= Rs15,600 - Rs15,000= Rs600
Rate= $\frac{I \times 100}{P \times T}$ %= $\frac{600 \times 100}{15,000 \times \frac{1}{2}}$ %= 8% Ans.
For 2nd half-year
P= Rs15,000; R=8% and T= ½ year
Interest= Rs $\frac{15,600 \times 8 \times \frac{1}{2}}{100}$ = Rs624
Amount= Rs15,600 + Rs624= Rs16,224
For 3rd half-year
P= Rs16,224; R=8% and T= ½ year
Interest= Rs $\frac{16,224 \times 8 \times \frac{1}{2}}{100}$ = Rs648.96
Amount= Rs16,224+ Rs648.96= Rs16,872.96 Ans.

Question 7

Ramesh invests Rs12,800 for three years at the rate of 10% per annum compound interest. Find:
(i)the sum due to Ramesh at the end of the first year.
(ii)the interest he earns for the second year.
(iii)the total amount due to him at the end of the third year.

Solution 7

For 1st year
P=Rs12,800; R=10% and T= 1year
Interest= Rs $\frac{12,800 \times 10 \times 1}{100}$ = Rs1,280
Amount= Rs12,800+ Rs1,280= Rs14,080
For 2nd year
P=Rs14,080; R=10% and T= 1 year
Interest= Rs $\frac{14,080 \times 10 \times 1}{100}$ = Rs1,408
Amount= Rs14,080+ Rs1,408= Rs15,488

For 3rd year
P=Rs15,488; R=10% and T= 1year

Interest= Rs $\frac{15,488 \times 10 \times 1}{100}$ = Rs1,548.80

Amount= Rs15,488+ Rs1,548.80= Rs17,036.80

Question 8

Rs8,000 is lent out at 7% compound interest for 2years. At the end of the first year Rs3,560 are returned. Calculate :
(i)the interest paid for the second year.
(ii)the total interest paid in two years.
(iii)the total amount of money paid in two years to clear the debt.

Solution 8

(i) For 1st year
P= Rs8,000; R=7% and T=1year

Interest= Rs $\frac{8,000 \times 7 \times 1}{100}$ = Rs560

Amount= Rs8,000+ Rs560= Rs8,560
Money returned= Rs3,560
Balance money for 2nd year= Rs8,560- Rs3,560= Rs5,000
For 2nd year
P= Rs5,000; R=7% and T=1year

Interest paid for the second year= Rs $\frac{5,000 \times 7 \times 1}{100}$ = Rs350 Ans.

(ii)The total interest paid in two years= Rs350 + Rs560
= Rs910 Ans.
(iii) The total amount of money paid in two years to clear the debt
= Rs8,000+ Rs910
= Rs8,910 Ans.

Question 9

The cost of a machine depreciated by Rs.4,000 during the first year and by Rs.3,600 during the second year. Calculate:

i. The rate of depreciation.
ii. The original cost of the machine.
iii. Its cost at the end of the third year.

Solution 9

(i)

Difference between depreciation in value between the first and second years
Rs.4,000 - Rs.3,600 = Rs.400
⇒ Depreciation of one year on Rs.4,000 = Rs.400

$$\Rightarrow \text{Rate of depreciation} = \frac{400}{4000} \times 100\% = 10\%$$

(ii)

Let Rs.100 be the original cost of the machine.

Depreciation during the 1st year = 10% of Rs.100 = Rs.10

When the values depreciates by Rs.10 during the 1st year, Original cost = Rs.100

⇒When the depreciation during 1st year = Rs.4,000,

$$\text{Original cost} = \frac{100}{10} \times 4000 = 40000$$

The original cost of the machine is Rs.40,000.

(iii)

Total depreciation during all the three years
= Depreciation in value during(1st year + 2nd year + 3rd year)
= Rs.4,000 + Rs.3,600 + 10% of (Rs.40,000 - Rs.7,600)
= Rs.4,000 + Rs.3,600 + Rs.3,240
= Rs.10,840

The cost of the machine at the end of the third year
= Rs.40,000 - Rs.10,840 = Rs.29,160

Question 10

Find the sum, invested at 10% compounded annually, on which the interest for the third year exceeds the interest of the first year by Rs252.

Solution 10

Let the sum of money be Rs 100
Rate of interest= 10%p.a.
Interest at the end of 1st year= 10% of Rs100= Rs10
Amount at the end of 1st year= Rs100 + Rs10= Rs110
Interest at the end of 2nd year=10% of Rs110 = Rs11
Amount at the end of 2nd year= Rs110 + Rs11= Rs121
Interest at the end of 3rd year=10% of Rs121= Rs12.10
∴ Difference between interest of 3rd year and 1st year
=Rs12.10- Rs10=Rs2.10
When difference is Rs2.10, principal is Rs100
When difference is Rs252, principal = $\frac{100 \times 252}{2.10}$ =Rs12,000 Ans.

Question 11

A man borrows Rs10,000 at 10% compound interest compounded yearly. At the end of each year, he pays back 30% of the sum borrowed. How much money is left unpaid just after the second year?

Solution 11

For 1st year
P= Rs10,000; R=10% and T= 1year
Interest= Rs $\frac{10,000 \times 10 \times 1}{100}$ =Rs1,000
Amount at the end of 1st year=Rs10,000+Rs1,000=Rs11,000
Money paid at the end of 1st year=30% of Rs10,000=Rs3,000

∴ Principal for 2nd year=Rs11,000- Rs3,000=Rs8,000
For 2nd year
P=Rs8,000; R=10% and T= 1year

$$\text{Interest= Rs } \frac{8,000 \times 10 \times 1}{100} = \text{Rs800}$$

Amount at the end of 2nd year=Rs8,000+Rs800= Rs8,800
Money paid at the end of 2nd year=30% of Rs10,000= Rs3,000
∴ Principal for 3rd year=Rs8,800- Rs3,000=Rs5,800 Ans.

Question 12

A man borrows Rs10,000 at 10% compound interest compounded yearly. At the end of each year, he pays back 20% of the amount for that year. How much money is left unpaid just after the second year?

Solution 12

For 1st year
P= Rs10,000; R=10% and T= 1year

$$\text{Interest= Rs } \frac{10,000 \times 10 \times 1}{100} = \text{Rs1,000}$$

Amount at the end of 1st year=Rs10,000+Rs1,000=Rs11,000
Money paid at the end of 1st year=20% of Rs11,000=Rs2,200
∴ Principal for 2nd year=Rs11,000- Rs2,200=Rs8,800
For 2nd year
P=Rs8,800; R=10% and T= 1year

$$\text{Interest= Rs } \frac{8,800 \times 10 \times 1}{100} = \text{Rs880}$$

Amount at the end of 2nd year=Rs8,800+Rs880= Rs9,680
Money paid at the end of 2nd year=20% of Rs9,680= Rs1,936
∴ Principal for 3rd year=Rs9,680- Rs1,936=Rs7,744 Ans.

Chapter 2 - Compound Interest (Without using formula)

Exercise Ex. 2(D)

Question 1

What sum will amount of ₹ 6,593.40 in 2 years at C.I., if the rates are 10 per cent and 11 per cent for the two successive years?

Solution 1

Let principal (p) = Rs. 100
For 1st year
P = Rs. 100
R = 10%
T = 1 year

$$I = \frac{100 \times 100 \times 1}{100} = \text{Rs.10}$$

A = 100 + 10 = Rs. 110
For 2nd year
P = Rs. 110
R = 11%
T = 1 year

$$I = \frac{110 \times 11 \times 1}{100} = Rs.12.10$$

A = 110 + 12.10 = Rs. 122.10
If Amount is Rs. 122.10 on a sum of Rs. = 100

If amount is Rs. 1, sum = $\frac{100}{122.10}$

If amount is Rs. 6593.40, sum = $\frac{100}{122.10} \times 6593.40$

= Rs. 5400

Question 2

The value of a machine depreciated by 10% per year during the first two years and 15% per year during the third year. Express the total depreciation of the machine, as per cent, during the three years.

Solution 2

Let the value of machine in the beginning= Rs. 100
For 1st year depreciation = 10% of Rs. 100 = Rs. 100
Value of machine for second year = 100 - 10
= Rs. 90
For 2nd year depreciation = 10% of 90 = Rs. 9
Value of machine for third year = 90 - 9
= Rs. 81
For 3rd year depreciation = 15% of 81
= Rs. 12.15
Value of machine at the end of third year = 81 - 12.15
= Rs. 68.85
Net depreciation = Rs. 100 - Rs. 68.85
= Rs. 31.15
Or 31.15%

Question 3

Rachna borrows Rs12,000 at 10 percent per annum interest compounded half-yearly. She repays Rs4,000 at the end of every six months. Calculate the third payment she has to make at end of 18 months in order to clear the entire loan.

Solution 3

For 1st half-year

P=Rs12,000; R=10% and T=1/2 year

Interest= Rs $\frac{12,000 \times 10 \times 1}{100 \times 2}$ = Rs600

Amount= RS12,000 + Rs600= Rs12,600

Money paid at the end of 1st half year=Rs4,000

Balance money for 2nd half-year= Rs12,600- Rs4,000=Rs8,600

For 2nd half-year

P=Rs8,600; R=10% and T=1/2 year

Interest=Rs $\frac{8,600 \times 10 \times 1}{100 \times 2}$ =Rs430

Amount= Rs8,600+ Rs430= Rs9,030

Money paid at the end of 2nd half-year=Rs4,000

Balance money for 3rd half-year= Rs9,030- Rs4,000=Rs5,030

For 3rd half-year

P=Rs5,030; R=10% and T=1/2 year

Interest = Rs $\frac{5,030 \times 10 \times 1}{100 \times 2}$ = Rs251.50

Amount= Rs5,030 + Rs251.50= Rs5,281.50

Question 4

On a certain sum of money, invested at the rate of 10 percent per annum compounded annually, the interest for the first year plus the interest for the third year is Rs2,652. Find the sum.

Solution 4

Let Principal= Rs 100

For 1st year

P=Rs100; R=10% and T=1year

Interest= Rs $\frac{100 \times 10 \times 1}{100}$ = Rs10

Amount= Rs100 + Rs10= Rs110

For 2nd year

P=Rs110; R=10% and T= 1year

Interest= Rs $\frac{110 \times 10 \times 1}{100}$ = Rs11

Amount= Rs110 + Rs11= Rs121

For 3rd year

P=Rs121; R=10% and T= 1year

Interest= Rs $\frac{121 \times 10 \times 1}{100}$ = Rs12.10

Sum of C.I. for 1st year and 3rd year=Rs10+Rs12.10=Rs22.10

When sum is Rs22.10, principal is Rs100

When sum is Rs2,652, principal =Rs $\frac{100 \times 2652}{22.10}$ =Rs12,000 Ans.

Question 5

During every financial year, the value of a machine depreciates by 12%. Find the original cost of a machine which depreciates by Rs2,640 during the second financial year of its purchase.

Solution 5

Let original value of machine=Rs100

For 1st year

P=Rs100; R=12% and T= 1year

Depreciation in 1st year= Rs $\frac{100 \times 12 \times 1}{100}$ =Rs12

Value at the end of 1st year=Rs100 - Rs12=Rs88

For 2nd year

P= Rs88; R=12% and T= 1year

Depreciation in 2nd year= Rs $\frac{88 \times 12 \times 1}{100}$ =Rs10.56

When depreciation in 2nd year is Rs10.56, original cost is Rs100

When depreciation in 2nd year is Rs2,640, original cost= $\frac{100 \times 2640}{10.56}$

=Rs25,000

Question 6

Find the sum on which the difference between the simple interest and compound interest at the rate of 8% per annum compounded annually would be Rs.64 in 2years.

Solution 6

Let Rs.x be the sum.

$$\text{Simple Interest(I)} = \frac{x \times 8 \times 1}{100} = 0.08x$$

Compound interest

For 1st year:

P = Rs.x, R = 8% and T=1

$$\Rightarrow \text{Interest(I)} = \frac{x \times 8 \times 1}{100} = 0.08x$$

For 2nd year:

P = Rs.x+Rs.0.08x = Rs.1.08x

$$\Rightarrow \text{Interest(I)} = \frac{1.08x \times 8 \times 1}{100} = 0.0864x$$

The difference between the simple interest and compound interest at the rate of 8% per annum compounded annually should be Rs.64 in 2 years.
⇒Rs.0.08x - Rs.0.0864x = Rs.64
⇒Rs.0.0064x = Rs.64
⇒x = Rs.10000

Hence the sum is Rs.10000.

Question 7

A sum of Rs13,500 is invested at 16% per annum compound interest for 5years.Calculate:
(i)the interest for the first year.
(ii)the amount at the end of first year.
(iii)the interest for the second year, correct to the nearest rupee.

Solution 7

For 1st year
P=Rs13,500; R=16% and T= 1year

$$\text{Interest= Rs } \frac{13,500 \times 16 \times 1}{100} = \text{Rs2,160}$$

Amount= Rs13,500 + Rs2,160= Rs15,660
For 2nd year
P=Rs15,660; R=16% and T= 1year

$$\text{Interest= Rs } \frac{15,660 \times 16 \times 1}{100} = \text{Rs2,505.60}$$

=Rs2,506

Question 8

Saurabh invests Rs48,000 for 7 years at 10% per annum compound interest.
Calculate:
(i)the interest for the first year.
(ii)the amount at the end of second year.
(iii)the interest for the third year.

Solution 8

For 1st year
P=Rs48,000; R=10% and T= 1year

$$\text{Interest= Rs } \frac{48,000 \times 10 \times 1}{100} = \text{Rs4,800}$$

Amount= Rs48,000+ Rs4,800= Rs52,800
For 2nd year
P=Rs52,800; R=10% and T= 1year

$$\text{Interest= Rs } \frac{52,800 \times 10 \times 1}{100} = \text{Rs5,280}$$

Amount= Rs52,800+ Rs5,280= Rs58,080
For 3rd year
P=Rs58,080; R=10% and T= 1year

$$\text{Interest= Rs } \frac{58,080 \times 10 \times 1}{100} = \text{Rs5,808}$$

Question 9

Ashok borrowed Rs.12,000 at some rate on compound interest. After a year, he paid back Rs.4,000. If the compound interest for the second year is Rs.920, find:

i. The rate of interest charged
ii. The amount of debt at the end of the second year

Solution 9

(i)

Let *x%* be the rate of interest charged.

For 1st year:

P = Rs.12,000, R = x% and T = 1

$$\Rightarrow \text{Interest(I)} = \frac{12000 \times x \times 1}{100} = 120x$$

For 2nd year:

After a year, Ashok paid back Rs.4,000.

P = Rs.12,000 + Rs.120x - Rs.4,000 = Rs.8,000 + Rs.120x

$$\Rightarrow \text{Interest(I)} = \frac{(8000 + 120x) \times x \times 1}{100} = \left(80x + 1.20x^2\right)$$

The compound interest for the second year is Rs.920

Rs. $(80x + 1.20x^2)$ = Rs.920

$\Rightarrow 1.20x^2 + 80x - 920 = 0$

$\Rightarrow 3x^2 + 200x - 2300 = 0$

$\Rightarrow 3x^2 + 230x - 30x - 2300 = 0$

$\Rightarrow x(3x + 230) - 10(3x + 230) = 0$

$\Rightarrow (3x + 230)(x - 10) = 0$

$\Rightarrow x = -230/3$ or $x = 10$

As rate of interest cannot be negative so x = 10.

Therefore the rate of interest charged is 10%.

(ii)

For 1st year:

Interest = Rs.120*x* = Rs.1200

For 2nd year:

Interest = Rs.$(80x + 1.20x^2)$ = Rs.920

The amount of debt at the end of the second year is equal to the addition of principal of the second year and interest for the two years.

Debt = Rs.8,000 + Rs.1200 + Rs.920 = Rs.10,120

Question 10

On a certain sum of money, lent out at C.I., interests for first, second and third years are Rs. 1,500; Rs. 1,725 and Rs. 2,070 respectively. Find the rate of interest for the (i) second year (ii) third year.

Solution 10

Total interest obtained in the first year = Rs. 1500

Interest for the second year – Total interest obtained in the first year

= Rs. 1,725 – Rs. 1,500

= Rs. 225

Rate of interest for the second year

$= \frac{\text{Rs. } 225}{\text{Rs. } 1,500} \times 100 = 15\%$

Interest for the third year – Interest for the second year

= Rs. 2,070 – Rs. 1,725

= Rs. 345

Rate of interest for the third year

$= \frac{\text{Rs. } 345}{\text{Rs. } 1,725} \times 100 = 20\%$

So, rate of interest for the second year and third year are 15% and 20% respectively.

Chapter 3 - Compound Interest (Using Formula)

Exercise Ex. 3(A)

Question 1

Find the amount and the compound interest on Rs12,000 in 3years at 5% compounded annually.

Solution 1

Given : P= Rs12,000; n=3years and r=5%

Amount= $P\left(1+\frac{r}{100}\right)^n = 12000\left(1+\frac{5}{100}\right)^3$

$= 12000\left(\frac{21}{20}\right)^3$

=Rs13,891.50 Ans.

∴ C.I. =RS13,891.50 - Rs12,000

= Rs1,891.50 Ans.

Question 2

Calculate the amount of Rs15,000 is lent at compound interest for 2years and the rates for the successive years are 8% and 10% respectively.

Solution 2

Given : P= Rs15,000; n=2years; r_1 =8% and r_2 =10%

Amount= $P\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right) = 15,000\left(1+\frac{8}{100}\right)\left(1+\frac{10}{100}\right)$

$= 15,000\left(\frac{27}{25}\right)\left(\frac{11}{10}\right)$

=Rs17,820 Ans.

Question 3

Calculate the compound interest accrued on Rs6,000 in 3years, compounded yearly, if the rates for the successive years are 5%, 8% and 10% respectively.

Solution 3

Given : P=Rs6,000; n= 3years; r_1= 5%; r_2= 8% and r_3 =10%

Amount= $P\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)\left(1+\frac{r_3}{100}\right)$

= $6,000\left(1+\frac{5}{100}\right)\left(1+\frac{8}{100}\right)\left(1+\frac{10}{100}\right)$

= $6000\left(\frac{21}{20}\right)\left(\frac{27}{25}\right)\left(\frac{11}{10}\right)$

=Rs7,484.40

∴ C.I. = Rs7,484.40 - Rs6,000 = Rs1,484.40 Ans.

Question 4

What sum of money will amount to Rs5,445 in 2years at 10% per annum compound interest?

Solution 4

Given : Amount= Rs5,445; n= 2years and r = 10%

∴ A= $P\left(1+\frac{r}{100}\right)^n$

⇒ 5,445= $P\left(1+\frac{10}{100}\right)^2$

⇒ 5,445= $P\left(\frac{11}{10}\right)^2$

⇒ P= $5,445\left(\frac{10}{11}\right)^2$ =Rs4,500 Ans.

Question 5

On what sum of money will the compound interest for 2years at 5% per annum amount to Rs768.75?

Solution 5

Given : C.I.= Rs768.75; n= 2years and r = 5%

∴ A= $P\left(1+\frac{r}{100}\right)^n$

⇒ A= $P\left(1+\frac{5}{100}\right)^2$

$\Rightarrow$ A= $P\left(\frac{21}{20}\right)^2 = \frac{441}{400}P$

$\therefore$ A - P =C.I

$\Rightarrow \frac{441}{400}P$ - P=Rs768.75

$\Rightarrow \frac{41}{400}P$ =Rs768.75

$\Rightarrow$ P=Rs $\frac{768.75 \times 400}{41} = Rs7,500$ Ans.

Question 6

Find the sum on which the compound interest for 3years at 10% per annum amounts to Rs1,655.

Solution 6

Given : C.I.= Rs1,655; n= 3years and r = 10%

$\therefore$ A= $P\left(1+\frac{r}{100}\right)^n$

$\Rightarrow$ A= $P\left(1+\frac{10}{100}\right)^3$

$\Rightarrow$ A= $P\left(\frac{11}{10}\right)^3 = \frac{1,331}{1,000}P$

$\therefore$ A - P =C.I

$\Rightarrow \frac{1,331}{1,000}P$ - P=Rs1,655

$\Rightarrow \frac{331}{1,000}P$ =Rs1,655

$\Rightarrow$ P=Rs $\frac{1,655 \times 1,000}{331} = Rs5,000$ Ans.

Question 7

What principal will amount to Rs9,856 in two years, if the rates of interest for successive years are 10% and 12% respectively?

Solution 7

Given : Amount =Rs9,856; n=2years; r_1 =10% and r_2 =12%

$$\therefore \text{Amount} = P\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)$$

$$\Rightarrow 9,856 = P\left(1+\frac{10}{100}\right)\left(1+\frac{12}{100}\right)$$

$$\Rightarrow 9,856 = P\left(\frac{11}{10}\right)\left(\frac{28}{25}\right)$$

$$\Rightarrow P = Rs\frac{9,856 \times 10 \times 25}{11 \times 28} = Rs8,000$$

Ans.

Question 8

On a certain sum, the compound interest in 2 years amounts to Rs.4,240. If the rate of interest for the successive years is 10% and 15% respectively, find the sum.

Solution 8

$$A = P\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)$$

$$\Rightarrow (P + 4240) = P\left(1+\frac{10}{100}\right)\left(1+\frac{15}{100}\right)$$

$$\Rightarrow (P + 4240) = P\,(1.265)$$

$$\Rightarrow P = 16000$$

The sum is Rs.16,000

Question 9

At what per cent per annum will Rs.6,000 amount to Rs.6,615 in 2 years when interest is compounded annually?

Solution 9

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$\Rightarrow 6,615 = 6,000\left(1+\frac{r}{100}\right)^2$$

$$\Rightarrow \left(1+\frac{r}{100}\right)^2 = \frac{6,615}{6,000}$$

$$\Rightarrow 1+\frac{r}{100} = \frac{21}{20}$$

$$\Rightarrow r = 5\%$$

At 5% per annum the sum of Rs.6,000 amounts to Rs.6,615 in 2 years when the interest is compounded annually.

Question 10

At what rate per cent compound interest, does a sum of money become 1.44 times of itself in 2years?

Solution 10

Let Principal = Rs y
Then Amount= Rs 1.44y
n= 2years

$$\therefore \quad A = P\left(1 + \frac{r}{100}\right)^n$$

$$\Rightarrow 1.44y = y\left(1 + \frac{r}{100}\right)^2$$

$$\Rightarrow \frac{1.44y}{y} = \left(1 + \frac{r}{100}\right)^2$$

$$\Rightarrow \frac{36}{25} = \left(1 + \frac{r}{100}\right)^2$$

$$\Rightarrow \left(\frac{6}{5}\right)^2 = \left(1 + \frac{r}{100}\right)^2$$

On comparing,

$$\frac{6}{5} = 1 + \frac{r}{100}$$

On solving, we get

$r = 20\%$

Question 11

At what rate per cent will a sum of Rs. 4,000 yield Rs. 1,324 as compound interest in 3 years?

Solution 11

Given: P = Rs. 4, 000, C.I. = Rs. 1, 324 and n = 3 years

Now, A = P + I

$\Rightarrow$ A = Rs. (4, 000 + 1, 324) = Rs. 5, 324

$$A = P\left(1 + \frac{r}{100}\right)^3$$

$$\Rightarrow 5324 = 4000\left(1 + \frac{r}{100}\right)^3$$

$$\Rightarrow \frac{5324}{4000} = \left(1 + \frac{r}{100}\right)^3$$

$$\Rightarrow \frac{1331}{1000} = \left(1 + \frac{r}{100}\right)^3$$

$$\Rightarrow \left(1 + \frac{R}{100}\right)^3 = \frac{1331}{1000} = \left(\frac{11}{10}\right)^3$$

$$\Rightarrow 1 + \frac{r}{100} = \frac{11}{10}$$

$$\Rightarrow \frac{r}{100} = \frac{11}{10} - 1 = \frac{1}{10}$$

$$\Rightarrow r = \frac{100}{100} = 10\%$$

Thus, the rate of interest is 10%.

Question 12

A person invests Rs5,000 for three years at a certain rate of interest compounded annually. At the end of two years this sum amounts to Rs6,272. Calculate :
(i)the rate of interest per annum.
(ii)the amount at the end of the third year.

Solution 12

Given: P=Rs5,000; A=Rs6,272 and n= 2years
(i)

$$\therefore \quad A = P\left(1 + \frac{r}{100}\right)^n$$

$$\Rightarrow 6,272 = 5,000\left(1+\frac{r}{100}\right)^2$$

$$\Rightarrow \frac{6,272}{5,000} = \left(1+\frac{r}{100}\right)^2$$

$$\Rightarrow \frac{784}{625} = \left(1+\frac{r}{100}\right)^2$$

$$\Rightarrow \left(\frac{28}{25}\right)^2 = \left(1+\frac{r}{100}\right)^2$$

On comparing

$$\frac{28}{25} = 1+\frac{r}{100}$$

On solving, we get

$r = 12\%$

(ii) Amount at the third year

$$= 5,000\left(1+\frac{12}{100}\right)^3$$

$$= 5,000\left(\frac{28}{25}\right)^3$$

$$= \text{Rs}7,024.64$$

Question 13

In how many years will Rs7,000 amount to Rs9,317 at 10% per annum compound interest?

Solution 13

Given : P=Rs7,000; A=Rs9,317 and r= 10%

$$\therefore \quad A = P\left(1+\frac{r}{100}\right)^n$$

$$\Rightarrow 9,317 = 7,000\left(1+\frac{10}{100}\right)^n$$

$$\Rightarrow \frac{9,317}{7,000} = \left(\frac{11}{10}\right)^n$$

$$\Rightarrow \frac{1,331}{1,000} = \left(\frac{11}{10}\right)^n$$

$$\Rightarrow \left(\frac{11}{10}\right)^3 = \left(\frac{11}{10}\right)^n$$

On comparing

$n = 3\text{years}$

Question 14

Find the time, in years, in which Rs4,000 will produce Rs630.50 as compound interest at 5% compounded annually.

Solution 14

Given : P=Rs4,000; C.I.=Rs630.50 and r=5%

$$\therefore \quad C.I. = P\left[\left(1+\frac{r}{100}\right)^n - 1\right]$$

$$\Rightarrow 630.50 = 4,000\left[\left(1+\frac{5}{100}\right)^n - 1\right]$$

$$\Rightarrow \frac{630.50}{4,000} = \left[\left(\frac{21}{20}\right)^n - 1\right]$$

$$\Rightarrow \frac{1,261}{8,000} = \left(\frac{21}{20}\right)^n - 1$$

$$\Rightarrow \frac{1,261}{8,000} + 1 = \left(\frac{21}{20}\right)^n$$

$$\Rightarrow \frac{9,261}{8,000} = \left(\frac{21}{20}\right)^n$$

$$\Rightarrow \left(\frac{21}{20}\right)^3 = \left(\frac{21}{20}\right)^n$$

On comparing

n = 3years

Question 15

Divide Rs28,730 between A and B so that when their shares are lent out at 10% compound interest compounded per year, the amount that A receives in 3years is the same as what B receives in 5years.

Solution 15

Let share of A = Rs y
share of B = Rs (28,730 - y)
rate of interest= 10%
According to question
Amount of A in 3years= Amount of B in 5years

$$\Rightarrow y\left(1+\frac{10}{100}\right)^3 = (28,730-y)\left(1+\frac{10}{100}\right)^5$$

$$\Rightarrow y = (28,730-y)\left(1+\frac{10}{100}\right)^2$$

$$\Rightarrow y=(28,730-y)\left(\frac{121}{100}\right)$$

$$\Rightarrow 100y=121(28,730-y)$$

$$\Rightarrow 100y+121y=121\times 28,730$$

$$\Rightarrow 221y=121\times 28,730$$

$$\Rightarrow y=\frac{121\times 28,730}{221} = Rs15,730$$

Therefore share of A=Rs15,730
Share of B=Rs28,730 - Rs 15,730=Rs13,000

Question 16

A sum of Rs44,200 is divided between John and Smith, 12years and 14years old respectively, in such a way that if their portions be invested at 10% per annum compound interest, they will receive equal amounts on reaching 16 years of age.
(i)What is the share of each out of Rs44,200?
(ii)What will each receive, when 16years old?

Solution 16

(i)Let share of John = Rs y
share of Smith = Rs (44,200 - y)
rate of interest= 10%
According to question
Amount of John in 4years= Amount of Smith in 2years

$$\Rightarrow y\left(1+\frac{10}{100}\right)^4 = (44,200-y)\left(1+\frac{10}{100}\right)^2$$

$$\Rightarrow y\left(1+\frac{10}{100}\right)^2 = (44,200-y)$$

$$\Rightarrow y\left(\frac{11}{10}\right)^2 =(44,200-y)$$

$$\Rightarrow 121y=100(44,200-y)$$

$$\Rightarrow 121y=100\times 44,200 - 100y$$

$$\Rightarrow 121y+100y=100\times 44,200$$

$$\Rightarrow 221y=100\times 44,200$$

$$\Rightarrow y=\frac{100\times 44,200}{221} = Rs20,000$$

Therefore share of John=Rs20,000
Share of Smith=Rs44,200- Rs 20,000=Rs24,200
(ii)Amount that each will receive

$$= 20,000\left(1+\frac{10}{100}\right)^4$$

$$= 20,000\left(\frac{11}{10}\right)^4$$

$$= Rs29,282$$

Question 17

The simple interest on a certain sum of money and at 10% per annum is Rs. 6,000 in 2 years, Find:

i. the sum.
ii. the amount due to the end of 3 years and at the same rate of interest compounded annually.
iii. the compound interest earned in 3 years.

Solution 17

(i) I = Rs. 6000, T = 2 years and R = 10%

$$\therefore P = \frac{I \times 100}{R \times T} = \frac{6000 \times 100}{10 \times 2} = Rs.\ 30,000$$

(ii) P = Rs. 30,000, n = 3 years and r = 10%

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$= 30000\left(1+\frac{10}{100}\right)^3$$

$$= 30000\left(\frac{11}{10}\right)^3$$

$$= 30000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10}$$

$$= Rs.\ 39,930$$

(iii) C.I. earned in 3 years = A − P = Rs. (39, 930 − 30, 000) = Rs. 930

Question 18

Find the difference between compound interest and simple interest on Rs. 8,000 in 2 years and at 5% per annum.

Solution 18

Given: P = Rs. 8000, R = 5%, T = 2 years

For simple interest,

$$S.I. = \frac{P \times R \times T}{100}$$

$$= \frac{8,000 \times 5 \times 2}{100}$$

$$= Rs.\ 800$$

For compound interest,

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$A = 8,000\left(1 + \frac{5}{100}\right)^2$$

$$= 8,000 \times \frac{21}{20} \times \frac{21}{20}$$

$$= Rs.8,820$$

$$C.I. = A - P$$

$$= Rs.\ (8,820 - 8,000)$$

$$= Rs.\ 820$$

Now, C.I. – S.I. = Rs. (820 – 800) = Rs. 20

Thus, the difference between the compound interest and the simple interest is Rs. 20.

Chapter 3 - Compound Interest (Using Formula)

Exercise Ex. 3(B)

Question 1

The difference between simple interest and compound interest on a certain sum is ₹ 54.40 for 2 years at 8 per cent per annum. Find the sum.

Solution 1

Let principal (P) = x

R = 8%

T = 2 years

$$SI = \frac{x \times 8 \times 2}{100} = \frac{4x}{25}$$

$$CI = A - P = x\left(1 + \frac{8}{100}\right)^2 - x$$

$$= x\left[\left(1 + \frac{2}{25}\right)^2 - 1\right]$$

$$= x\left[\left(\frac{27}{25}\right)^2 - 1\right]$$

$$= \frac{104}{625}x$$

Given, CI = SI = 54.40

$$\frac{104x}{625} - \frac{4x}{25} = Rs.54.40$$

$$x\left(\frac{104}{625} - \frac{4}{25} \times \frac{25}{25}\right) = 54.40$$

$$x\left(\frac{4}{625}\right) = 54.40$$

$$x = \frac{54.40 \times 625}{4}$$

$$x = Rs.8500$$

Thus, principalsum = Rs. 8500

Question 2

A sum of money, invested at compound interest, amounts to ₹ 19,360 in 2 years and to ₹ 23,425.60 in 4 years. Find the rate per cent and the original sum of money.

Solution 2

(for 2 years) A = Rs. 19360
T = 2 years
Let P = X

$$\times\left(1+\frac{R}{100}\right)^2 = 19360 \quad ...(1)$$

A (for 4 years) = Rs. 23425.60

$$\times\left(1+\frac{R}{100}\right)^4 = 23425.60 \quad ...(2)$$

(2) ÷ (1)

$$\left(1+\frac{R}{100}\right)^2 = \frac{23425.60}{19360}$$

$$\left(1+\frac{R}{100}\right)^2 = \frac{2342560}{1936000}$$

$$\left(1+\frac{R}{100}\right)^2 = \frac{14641}{12100}$$

$$\left(1+\frac{R}{100}\right)^2 = \left(\frac{121}{110}\right)^2$$

$$1+\frac{R}{100} = \frac{121}{110}$$

$$\frac{R}{100} = \frac{121}{110} - 1$$

$$R = 10\%$$

$$x\left(1+\frac{10}{100}\right)^2 = 19360$$

Form (1)

$$x = \frac{19360 \times 10 \times 10}{11 \times 11}$$

$$x = \text{Rs. } 16000$$

Thus, sum = Rs. 16000

Question 3

A sum of money lent out at C.I. at a certain rate per annum becomes three times of itself in 8 years. Find in how many years will the money becomes twenty-seven times of itself at the same rate of interest p.a.

Solution 3

Let principal = x, A = 3x, T = 8 years, R = ?
Case I,

$$A = P\left(1+\frac{R}{100}\right)^T$$

$$3x = x\left(1+\frac{R}{100}\right)^8$$

$$3^{y/8} = 1+\frac{R}{100} \qquad \ldots(1)$$

Case II,
P = x,A = 27x,T = ?

$$27x = x\left(1+\frac{R}{100}\right)^T$$

$$27^{1/T} = 1 + \frac{R}{100} \qquad \ldots(2)$$

From (1) and (2) $3^{1/8} = 27^{1/T}$

$3^{1/8} = 3^{\frac{1}{8}} = 3/T$

$T = 24$

Time = 24 years.

Question 4

On what sum of money will compound interest (payable annually) for 2 years be the same as simple interest on ₹ 9,430 for 10 years, both at the rate of 5 per cent per annum?

Solution 4

P = Rs. 9430
R = 5%
R = 10 years

SI = $\frac{9430 \times 5 \times 10}{100} = Rs.4715$

Let sum = x
CI = 4715, T = 2 years, Rs= 5%
CI = AP

$$4715 = x\left(1+\frac{R}{100}\right)^{T} - x$$

$$4715 = x\left(1+\frac{5}{100}\right)^{2} - x$$

$$4715 = x\left[\left(\frac{21}{20}\right)^{2} - 1\right]$$

$$4715 = x\left(\frac{441-400}{400}\right)$$

$$x = \frac{4715 \times 400}{41} = \text{Rs. } 46,000$$

Thus principal from = Rs. 46,000

Question 5

Kamal and Anand each lent the same sum of money for 2 years at 5% at simple interest and compound interst respectively. Anand recived ₹ 15 more than Kamal. Find the amount of money lent by each and the interest received.

Solution 5

Let principal = Rs. 100, R = 5% T = 2 years

For Kamal, SI = $\frac{100 \times 5 \times 2}{100} = \text{Rs. } 10$

For Anand, $A = P\left(1+\frac{R}{100}\right)^{T}$

$$= 100\left(1+\frac{5}{100}\right)^{2}$$

$$= 100 \times \frac{21}{20} \times \frac{21}{20}$$

$$= \frac{441}{4}$$

CI = $\frac{441}{4} - 100 = \frac{41}{4}$

Difference of CI and SI = $\frac{41}{4} - 10$

$= \frac{41 - 40}{4}$

$= \text{Rs.}\ \frac{1}{4}$

When difference is Rs. $\frac{1}{4}$, then principal = Rs. 100
If difference is 1, principal = 100 $\times$ 4
If difference is Rs, 15, principal = 100 $\times$ 4 $\times$ 15 = Rs. 6000

For kamal, interest = $\frac{6000 \times 5 \times 2}{100} = \text{Rs.}600$

For Anand, interest = $6000\left(1 + \frac{5}{100}\right)^2 - 6000$

$= 6000\left(\left(\frac{21}{20}\right)^2 - 1\right)$

$= 6000\left[\frac{441}{400} - 1\right]$

$= 6000 \times \frac{41}{400}$

$= \text{Rs.}615$

Question 6

Simple interest on a sum of money for 2 years at 4% is ₹ 450. Find compound interest of the same sum and at the same rate for 2 years.

Solution 6

SI = Rs. 450
R = 4%
R = 2 years
P = ?

$$P = \frac{SI \times 100}{R \times T} = \frac{450 \times 100}{4 \times 2} = Rs.5625$$

Now, P = 5625, R = 4%, T = 2 years

$$A = 5625\left(1 + \frac{4}{100}\right)^2 = 5625\left(\frac{26}{25}\right)^2$$

$$= \frac{3802500}{625} = Rs.6084$$

CI = A - P = 6084 - 5625
= Rs. 459

Question 7

Simple interest on a certain sum of money for 4 years at 4% per annum exceeds the compound interest on the same sum for 3 years at 5 per cent per annum by ₹ 228. Find the sum.

Solution 7

Let principal (P), R = 4%, T = 4 years

$$SI = \frac{P \times 4 \times 4}{100} = \frac{4P}{25}$$

$$CI = P\left(1 + \frac{5}{100}\right) - P = P\left[\left(\frac{21}{20}\right)^3 - 1\right] = P\left(\frac{9261}{8000} - 1\right)$$

$$= \frac{1261}{8000}P$$

Given SI -; CI = Rs. 228

$$\frac{4P}{25} - \frac{1261}{8000}P = 228$$
$$\frac{4 \times 320P - 1261P}{8000} = 228$$
$$19P = 228 \times 8000$$
$$P = \frac{228 \times 8000}{19} = Rs.96000$$

Thus, Principal = Rs. 96000

Question 8

Compound interest on a certain sum of money at 5% per annum for two years is ₹ 246. Calculate simple interest on the same sum for 3 years at 6% per annum.

Solution 8

CI = Rs. 246, R = 5%, T = 2 years
CI = A - P

$$246 = P\left(1 + \frac{5}{100}\right)^2 - P$$
$$246 = P\left[\left(\frac{21}{20}\right)^2 - 1\right]$$
$$246 = P\frac{61}{400}$$
$$P = \frac{246 \times 400}{41}$$
$$= Rs.2400$$

Now, P = Rs. 2400 ,R = 6%, T = 3 years

$$SI = \frac{2400 \times 6 \times 3}{100}$$
$$= Rs.432$$

Question 9

A certain sum of money amounts to Rs. 23,400 in 3 years at 10% per annum simple interest. Find the amount of the same sum in 2 years and at 10% p.a. compound interest.

Solution 9

Let the sum (principle) = x
Given Amount = 23400, R = 10% and T = 3 years

$$\Rightarrow \text{interest I} = \frac{x \times 10 \times 3}{100} = \frac{3x}{10}$$

Amount = Principle + Interest

$$23400 = x + \frac{3x}{10}$$

$x = 18000$

Principle = 18000

Now,

Principle = `18000, r = 10% and n = 2 years

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$A = 18000\left(1 + \frac{10}{100}\right)^2$$

$$A = 18000\left(\frac{11}{10}\right)^2$$

$$A = 18000\left(\frac{121}{100}\right)$$

$$A = 21780$$

The amount of the same sum in 2 years and at 10% p.a. compound interest is 21780.

Question 10

Mohit borrowed a certain sum at 5% per annum compound interest and cleared this loan by paying Rs. 12,600 at the end of the first year and Rs. 17,640 at the end of the second year. Find the sum borrowed.

Solution 10

For the payment of Rs. 12,600 at the end of first year:

$A = \text{Rs. } 12,600;\ n = 1 \text{ year and } r = 5\%$

$$\text{Now, } A = P\left(1 + \frac{r}{100}\right)^n$$

$$\Rightarrow 12,600 = P\left(1 + \frac{5}{100}\right)^1$$

$$\Rightarrow 12,600 = P\left(\frac{21}{20}\right)$$

$$\Rightarrow P = \frac{20}{21} \times 12,600 = \text{Rs. } 12,000$$

For the payment of Rs. 17,640 at the end of second year:

$A = \text{Rs. } 17,640;\ n = 2 \text{ years and } r = 5\%$

$$\text{Now, } A = P\left(1 + \frac{r}{100}\right)^n$$

$$\Rightarrow 17,640 = P\left(1 + \frac{5}{100}\right)^2$$

$$\Rightarrow 17,640 = P\left(\frac{21}{20}\right)^2$$

$$\Rightarrow P = \frac{20}{21} \times \frac{20}{21} \times 17,640 = \text{Rs. } 16,000$$

$\therefore$ Sum borrowed = Rs. $(12,000 + 16,000)$ = Rs. 28,000

Chapter 3 - Compound Interest (Using Formula)

Exercise Ex. 3(C)

Question 1

If the interest is compounded half-yearly, calculate the amount when principal is Rs7,400; the rate of interest is 5% per annum and the duration is one year.

Solution 1

Given: P=Rs7,400; r=5% p.a. and n= 1year
Since the interest is compounded half-yearly,

Then $A=P\left(1+\frac{r}{2\times 100}\right)^{n\times 2}$

$=7,400\left(1+\frac{5}{2\times 100}\right)^{1\times 2}$

$=7,400\left(\frac{41}{40}\right)^{2}$

$=Rs7,774.63$

Question 2

Find the difference between the compound interest compounded yearly and half-yearly on Rs10,000 for 18 months at 10% per annum.

Solution 2

(i)When interest is compounded yearly

Given: P=Rs10,000; n=18months= $1\frac{1}{2}$ year and r=10%p.a.

For 1year

$A=P\left(1+\frac{r}{100}\right)^{n}=10,000\left(1+\frac{10}{100}\right)^{1}=10,000\left(\frac{11}{10}\right)^{1}=Rs11,000$

For1/2 year

P=Rs11,000;n= 1/2 year and r=10%

$A=P\left(1+\frac{r}{2\times 100}\right)^{n\times 2}=11,000\left(1+\frac{10}{2\times 100}\right)^{\frac{1}{2}\times 2}=11,000\left(\frac{21}{20}\right)^{1}$

$=Rs11,550$

∴ C.I.= Rs11,550 - Rs10,000= Rs1,550

(ii)When interest is compounded half-yearly

P=Rs10,000; n= $1\frac{1}{2}$ year and r=10%p.a.

$A=P\left(1+\frac{r}{2\times 100}\right)^{n\times 2}=10,000\left(1+\frac{10}{2\times 100}\right)^{\frac{3}{2}\times 2}$

$=10,000\left(\frac{21}{20}\right)^{3}$

$=Rs11,576.25$

∴ C.I.= Rs11,576.25 - Rs10,000=Rs1,576.25

∴ Difference between both C.I.= Rs1,576.25 - Rs1,550

= Rs26.25 Ans.

Question 3

A man borrowed Rs.16,000 for 3 years under the following terms:

20% simple interest for the first 2 years.
20% C.I. for the remaining one year on the amount due after 2 years, the interest being compounded half-yearly.
Find the total amount to be paid at the end of the three years.

Solution 3

For the first 2 years

$$S.I. = \frac{P \times N \times R}{100}$$
$$\Rightarrow S.I. = \frac{16,000 \times 2 \times 20}{100}$$
$$\Rightarrow S.I. = 6,400$$

$$Amount = S.I. + P$$
$$\Rightarrow Amount = 6,400 + 16,000$$
$$\Rightarrow Amount = 22,400$$

Amount in the account at the end of the two years is Rs.22,400.

For the remaining one year

$$A = P\left(1 + \frac{r}{2 \times 100}\right)^{n \times 2}$$
$$\Rightarrow A = 22,400\left(1 + \frac{20}{200}\right)^{2}$$
$$\Rightarrow A = 22,400\left(\frac{11}{10}\right)^{2}$$
$$\Rightarrow A = 27,104$$

The total amount to be paid at the end of the three years is Rs.27,104.

Question 4

What sum of money will amount to Rs.27,783 in one and a half years at 10% per annum compounded half yearly?

Solution 4

$$A = P\left(1 + \frac{r}{2 \times 100}\right)^{n \times 2}$$

$$\Rightarrow 27,783 = P\left(1 + \frac{10}{200}\right)^{\frac{3}{2} \times 2}$$

$$\Rightarrow 27,783 = P\left(\frac{21}{20}\right)^{3}$$

$$\Rightarrow P = 27,783\left(\frac{20}{21}\right)^{3}$$

$$\Rightarrow P = 24,000$$

The sum of Rs.24,000 amount Rs.27,783 in one and a half years at 10% per annum compounded half yearly.

Question 5

Ashok invests a certain sum of money at 20% per annum, compounded yearly. Geeta invests an equal amount of money at the same rate of interest per annum compounded half-yearly. If Geeta gets Rs33 more than Ashok in 18 months, calculate the money invested.

Solution 5

(i)For Ashok(interest is compounded yearly)

Let P=Rs y; n=18months= $1\frac{1}{2}$ year and r=20%p.a.

For 1year

$$A=P\left(1+\frac{r}{100}\right)^{n} = y\left(1+\frac{20}{100}\right)^{1} = \left(\frac{6}{5}\right)y$$

For1/2 year

P $= Rs\left(\frac{6}{5}\right)y$;n= ½ year and r=20%

$$A=P\left(1+\frac{r}{2\times 100}\right)^{n\times 2} = Rs\left(\frac{6}{5}\right)y\left(1+\frac{20}{2\times 100}\right)^{\frac{1}{2}\times 2} = Rs\left(\frac{66}{50}\right)y$$

(ii)For Geeta(interest is compounded half-yearly)

P=Rs y; n= $1\frac{1}{2}$ year and r=20%p.a.

$$A=P\left(1+\frac{r}{2\times 100}\right)^{n\times 2} = y\left(1+\frac{20}{2\times 100}\right)^{\frac{3}{2}\times 2} = y\left(\frac{11}{10}\right)^{3}$$

$= Rs\left(\frac{1,331}{1,000}\right)y$

According to question

$\therefore \left(\frac{1,331}{1,000}\right)y - \left(\frac{66}{50}\right)y = Rs33$

$\Rightarrow \left(\frac{11}{1,000}\right)y = Rs33$

$\Rightarrow y = Rs\frac{33 \times 1,000}{11} = Rs3,000$

$\therefore$ Money invested by each person=Rs3,000 Ans.

Question 6

At what rate of interest per annum will a sum of Rs.62,500 earn a compound interest of Rs.5,100 in one year? The interest is to be compounded half yearly.

Solution 6

$$C.I = P\left[\left(1 + \frac{r}{2 \times 100}\right)^{2 \times n} - 1\right]$$

$$\Rightarrow 5,100 = 62,500\left[\left(1 + \frac{r}{200}\right)^{2} - 1\right]$$

$$\Rightarrow \left(1 + \frac{r}{200}\right)^{2} = \frac{67,600}{62,500}$$

$$\Rightarrow 1 + \frac{r}{200} = \frac{260}{250}$$

$$\Rightarrow r = 8$$

The rate of interest is 8%.

Question 7

In what time will Rs1,500 yield Rs496.50 as compound interest at 20% per year compounded half-yearly?

Solution 7

Given: P=Rs1,500; C.I.=Rs496.50 and r=20%

Since interest is compounded semi-annually

Then $$C.I. = P\left[\left(1 + \frac{r}{2 \times 100}\right)^{n \times 2} - 1\right]$$

$$\Rightarrow 496.50 = 1,500\left[\left(1+\frac{20}{2\times 100}\right)^{n\times 2}-1\right]$$

$$\Rightarrow \frac{496.50}{1,500} = \left(\frac{11}{10}\right)^{2n}-1$$

$$\Rightarrow \frac{331}{1,000}+1 = \left(\frac{11}{10}\right)^{2n}$$

$$\Rightarrow \frac{1,331}{1,000} = \left(\frac{11}{10}\right)^{2n}$$

$$\Rightarrow \left(\frac{11}{10}\right)^{3} = \left(\frac{11}{10}\right)^{2n}$$

On comparing, we get

$$2n=3 \quad \Rightarrow \quad n=1\frac{1}{2}\text{ years}$$

Ans.

Question 8

Calculate the C.I. on Rs3,500 at 6% per annum for 3years, the interest being compounded half-yearly. Do not use mathematical tables. Use the necessary information from the following:
$(1.06)^3$ =1.191016; $(1.03)^3$ = 1.092727
$(1.06)^6$ =1.418519; $(1.03)^6$ = 1.194052

Solution 8

Given: P=Rs 3,500; r=6% and n= 3years
Since interest is being compounded half-yearly

$$\text{C.I.} = P\left[\left(1+\frac{r}{2\times 100}\right)^{n\times 2}-1\right]$$

Then

$$= 3,500\left[\left(1+\frac{6}{2\times 100}\right)^{3\times 2}-1\right]$$

$$= 3,500\left[\left(\frac{103}{100}\right)^{6}-1\right]$$

$$= 3,500\left[(1.03)^{6}-1\right]$$

$$= 3,500\left[1.194052-1\right]$$

$$= 3,500\times 0.194052$$

$$= \text{Rs}679.18$$

Ans.

Question 9

Find the difference between compound interest and simple interest on Rs12,000 and in $1\frac{1}{2}$ years at 10% compounded yearly.

Solution 9

Given: P=Rs12,000; n= $1\frac{1}{2}$ years and r= 10%

$$S.I.=\frac{P\times R\times T}{100}=\frac{12,000\times 10\times \frac{3}{2}}{100}=Rs1,800$$

To calculate C.I.

For 1year

P=Rs12,000; n=1year and r=10%

$$A=P\left(1+\frac{r}{100}\right)^{n}=12,000\left(1+\frac{10}{100}\right)^{1}=Rs13,200$$

For next 1/2 year

P=Rs13,200;n= 1/2 year and r=10%

$$A=P\left(1+\frac{r}{2\times 100}\right)^{n\times 2}=13,200\left(1+\frac{10}{2\times 100}\right)^{\frac{1}{2}\times 2}$$

$$=13,200\left(\frac{21}{20}\right)^{1}$$

$$=Rs13,860$$

∴ C.I.=Rs13,860 - Rs12,000= Rs1,860

∴ Difference between C.I. and S.I

=Rs1,860 - Rs1,800=Rs60 Ans.

Question 10

Find the difference between compound interest and simple interest on Rs12,000 and in $1\frac{1}{2}$ years at 10% compounded half-yearly.

Solution 10

Given: P=Rs12,000; n= $1\frac{1}{2}$ years and r= 10% $S.I.=\frac{P\times R\times T}{100}=\frac{12,000\times 10\times \frac{3}{2}}{100}=Rs1,800$

To calculate C.I.(compounded half-yearly)

P=Rs12,000;n= $1\frac{1}{2}$ years and r=10%

$$A=P\left(1+\frac{r}{2\times 100}\right)^{n\times 2}=12,000\left(1+\frac{10}{2\times 100}\right)^{\frac{3}{2}\times 2}$$

$$=12,000\left(\frac{21}{20}\right)^{3}$$

$$=Rs13,891.50$$

∴ C.I. $=Rs13,891.50$ - Rs12,000= Rs1,891.50

∴ Difference between C.I. and S.I

=Rs1,891.50 - Rs1,800=Rs91.50 Ans.

Chapter 3 - Compound Interest (Using Formula)

Exercise Ex. 3(D)

Question 1

The cost of a machine is supposed to depreciate each year at 12% of its value at the beginning of the year. If the machine is valued at Rs44,000 at the beginning of 2008, find its value :
(i)at the end of 2009.
(ii)at the beginning of 2007.

Solution 1

Cost of machine in 2008 = Rs44,000
Depreciation rate=12%
(i) $\therefore$ Cost of machine at the end of 2009

$$= P\left(1-\frac{r}{100}\right)^n$$

$$= 44,000\left(1-\frac{12}{100}\right)^2$$

$$= 44,000 \times \left(\frac{88}{100}\right)^2 = \text{Rs}34,073.60 \quad \text{Ans.}$$

(ii) Cost of machine at the beginning of 2007(P)

$$A = P\left(1-\frac{r}{100}\right)^n$$

$$\Rightarrow 44,000 = P\left(1-\frac{12}{100}\right)^1$$

$$\Rightarrow 44,000 = P\left(\frac{88}{100}\right)^1$$

$$\Rightarrow P = \frac{44,000 \times 100}{88} = \text{Rs}50,000 \quad \text{Ans}$$

Question 2

The value of an article decreases for two years at the rate of 10% per year and then in the third year it increases by 10%. Find the original value of the article, if its value at the end of 3 years is Rs.40,095.

Solution 2

Let x be the value of the article.

The value of an article decreases for two years at the rate of 10% per year.

The value of the article at the end of the 1st year is
X - 10% of x = 0.90x

The value of the article at the end of the 2nd year is
0.90x - 10% of (0.90x) = 0.81x

The value of the article increases in the 3rd year by 10%.

The value of the article at the end of 3rd year is
0.81x + 10% of (0.81x) = 0.891x

The value of the article at the end of 3 years is Rs.40,095.

$$0.891x = 40,095$$

$$\Rightarrow x = 45,000$$

The original value of the article is Rs.45,000.

Question 3

According to a census taken towards the end of the year 2009, the population of a rural town was found to be 64,000. The census authority also found that the population of this particular town had a growth of 5% per annum. In how many years after 2009 did the population of this town reach 74,088 ?

Solution 3

Population in 2009 (P) = 64,000
Let after n years its population be 74,088(A)
Growth rate= 5% per annum

$$\therefore \quad A = P\left(1 + \frac{r}{100}\right)^n$$

$$\Rightarrow 74,088 = 64,000\left(1 + \frac{5}{100}\right)^n$$

$$\Rightarrow \frac{74,088}{64,000} = \left(\frac{21}{20}\right)^n$$

$$\Rightarrow \frac{9,261}{8,000} = \left(\frac{21}{20}\right)^n$$

$$\Rightarrow \left(\frac{21}{20}\right)^3 = \left(\frac{21}{20}\right)^n$$

On comparing, we get

$$n = 3 \text{ years}$$

Ans.

Question 4

The population of a town decreased by 12% during 1998 and then increased by 8% during 1999. Find the population of the town, at the beginning of 1998, if at the end of 1999 its population was 2,85,120.

Solution 4

Let the population in the beginning of 1998 = P
The population at the end of 1999 = 2,85,120(A)
r_1 = - 12% and r_2 = +8%

$$\therefore \quad A = P\left(1 - \frac{r_1}{100}\right)\left(1 + \frac{r_2}{100}\right)$$

$$\Rightarrow 2,85,120 = P\left(1 - \frac{12}{100}\right)\left(1 + \frac{8}{100}\right)$$

$$\Rightarrow 2,85,120 = P\left(\frac{22}{25}\right)\left(\frac{27}{25}\right)$$

$$\Rightarrow P = \frac{2,85,120 \times 25 \times 25}{22 \times 27} = 3,00,000$$ Ans.

Question 5

A sum of money, invested at compound interest, amounts to Rs 16,500 in 1 year and to Rs19,965 in 3 years. Find the rate per cent and the original sum of money invested.

Solution 5

Let sum of money be Rs P and rate of interest= r%
Money after 1year= Rs16,500
Money after 3years=Rs19,965
For 1year

$$\therefore \quad A = P\left(1 + \frac{r}{100}\right)^n$$

$$\Rightarrow 16,500 = P\left(1 + \frac{r}{100}\right)^1 \text{----------(1)}$$

For 3years

$$\therefore \quad A = P\left(1 + \frac{r}{100}\right)^n$$

$$\Rightarrow 19,965 = P\left(1 + \frac{r}{100}\right)^3 \text{----------(2)}$$

Divide eq^n (2) by eq^n (1)

$$\frac{19,965}{16,500} = \frac{P\left(1 + \frac{r}{100}\right)^3}{P\left(1 + \frac{r}{100}\right)^1}$$

$$\Rightarrow \frac{121}{100} = \left(1 + \frac{r}{100}\right)^2$$

$$\Rightarrow \left(\frac{11}{10}\right)^2 = \left(1 + \frac{r}{100}\right)^2$$

On comparing, we get

$\frac{11}{10} = 1 + \frac{r}{100} \Rightarrow$ r= 10% Ans.

Put value of r in eq^n (1)

$$16,500 = P\left(1 + \frac{10}{100}\right)$$

$$\Rightarrow P = \frac{16,500 \times 10}{11} = \text{Rs}15,000$$ Ans

Question 6

The difference between C.I. and S.I. on Rs7,500 for two years is Rs12 at the same rate of interest per annum. Find the rate of interest.

Solution 6

Given: P = Rs7,500 and Time(n)= 2years

Let rate of interest = y%

$$\therefore \text{ S.I.} = \frac{P \times R \times T}{100} = \frac{7,500 \times y \times 2}{100} = \text{Rs}150y$$

$$\therefore \text{ C.I.} = P\left(1 + \frac{r}{100}\right)^n - P = \text{Rs}7,500\left(1 + \frac{y}{100}\right)^2 - \text{Rs}7,500$$

Given: C.I. -; S.I. = Rs12

$$\Rightarrow 7,500\left(1 + \frac{y}{100}\right)^2 - 7,500 - 150y = 12$$

$$\Rightarrow 7,500\left(1 + \frac{y^2}{10000} + \frac{2y}{100}\right) - 7,500 - 150y = 12$$

$$\Rightarrow 7,500 + \frac{7,500y^2}{10000} + 150y - 7,500 - 150y = 12$$

$$\Rightarrow \frac{3y^2}{4} = 12$$

$\Rightarrow y^2 = 16 \quad \Rightarrow y = 4\%$ Ans.

Question 7

A sum of money lent out at C.I. at a certain rate per annum becomes three times of itself in 10years. Find in how many years will the money become twenty-seven times of itself at the same rate of interest p.a.

Solution 7

Let Principal be Rs y and rate= r%

According to 1st condition

Amount in 10 years = Rs 3y

$$\therefore\ A=P\left(1+\frac{r}{100}\right)^n$$

$$\Rightarrow 3y=y\left(1+\frac{r}{100}\right)^{10}$$

$$\Rightarrow 3=\left(1+\frac{r}{100}\right)^{10} \text{ -------(1)}$$

According to 2nd condition

Let after n years amount will be Rs 27y

$$\therefore\ A=P\left(1+\frac{r}{100}\right)^n$$

$$\Rightarrow 27y=y\left(1+\frac{r}{100}\right)^n$$

$$\Rightarrow (3)^3=\left(1+\frac{r}{100}\right)^n$$

Put value from first equation

$$\Rightarrow \left[\left(1+\frac{r}{100}\right)^{10}\right]^3=\left(1+\frac{r}{100}\right)^n$$

On comparing, we get

n = 10 x 3= 30years Ans.

Question 8

Mr. Sharma borrowed a certain sum of money at 10% per annum compounded annually. If by paying Rs.19,360 at the end of the second year and Rs.31,944 at the end of the third year he clears the debt; find the sum borrowed by him.

Solution 8

At the end of the two years the amount is

$$A_1=P\left(1+\frac{r}{100}\right)^n$$

$$\Rightarrow A_1=P\left(1+\frac{10}{100}\right)^2$$

Mr. Sharma paid Rs.19,360 at the end of the second year.
So for the third year the principal is A_1 - 19,360.
Also he cleared the debt by paying Rs.31,944 at the end of the third year.

$$A_2 = P\left(1+\frac{r}{100}\right)^n$$

$$\Rightarrow 31,944 = \left(P\left(1+\frac{10}{100}\right)^2 - 19,360\right)\left(1+\frac{10}{100}\right)^1$$

$$\Rightarrow 29040 = \left(P\left(1+\frac{10}{100}\right)^2 - 19,360\right)$$

$$\Rightarrow P\left(1+\frac{10}{100}\right)^2 = 48,400$$

$$\Rightarrow P = 40,000$$

Mr. Sharma borrowed Rs.40,000.

Question 9

The difference between compound interest for a year payable half-yearly and simple interest on a certain sum of money lent out at 10% for a year is Rs15. Find the sum of money lent out.

Solution 9

Let sum of money be RS y

To calculate S.I.

$$S.I. = \frac{P \times R \times T}{100} = \frac{y \times 10 \times 1}{100} = Rs\,\frac{y}{10}$$

To calculate C.I.(compounded half-yearly)

$$\therefore \quad C.I. = P\left[\left(1+\frac{r}{2\times 100}\right)^{n\times 2} - 1\right] = y\left[\left(1+\frac{10}{2\times 100}\right)^{1\times 2} - 1\right]$$

$$= y\left[\left(\frac{21}{20}\right)^2 - 1\right] = \left(\frac{41}{400}\right)y$$

Given : C.I. - S.I = Rs15

$$\Rightarrow \left(\frac{41}{400}\right)y - \frac{y}{10} = 15$$

$$\Rightarrow \frac{y}{400} = 15 \Rightarrow y = Rs6,000 \qquad \text{Ans.}$$

Question 10

The ages of Pramod and Rohit are 16 years and 18 years respectively. In what ratio must they invest money at 5% p.a. compounded yearly so that both get the same sum on attaining the age of 25 years?

Solution 10

Let Rs.x and Rs.y be the money invested by Pramod and Rohit respectively such that they will get the same sum on attaining the age of 25 years.

Pramod will attain the age of 25 years after 25 - 16 = 9 years

Rohit will attain the age of 25 years after 25 -18 = 7 years

$$x\left(1+\frac{5}{100}\right)^{9} = y\left(1+\frac{5}{100}\right)^{7}$$

$$\Rightarrow \frac{x}{y} = \frac{1}{\left(1+\frac{5}{100}\right)^{2}}$$

$$\Rightarrow \frac{x}{y} = \frac{400}{441}$$

Pramod and Rohit should invest in 400:441 ratio respectively such that they will get the same sum on attaining the age of 25 years.

Chapter 3 - Compound Interest (Using Formula)

Exercise Ex. 3(E)

Question 1

Simple interest on a sum of money for 2 years at 4% is Rs.450. Find compound interest on the same sum and at the same rate for 1 year, if the interest is reckoned half yearly.

Solution 1

1st case

Given: S.I. = Rs 450;Time= 2 years and Rate = 4%

$$\therefore \quad \text{Principal} = \frac{I \times 100}{R \times T} = \frac{450 \times 100}{4 \times 2} = \text{Rs. } 5625$$

2nd case(compounded half-yearly)

P = Rs.5,625;n= 1 year and r = 4%

$$\therefore \quad A = P\left(1+\frac{r}{2 \times 100}\right)^{n \times 2} = 5{,}625\left(1+\frac{4}{2 \times 100}\right)^{1 \times 2}$$

$$= 5{,}625\left(\frac{51}{50}\right)^{2} = \text{Rs. } 5852.25$$

$$\therefore \quad \text{C.I.} = 5{,}852.25 - 5{,}625 = \text{Rs. } 227.25$$

Question 2

Find the compound interest to the nearest rupee on Rs. 10,800 for $2\frac{1}{2}$ years at 10% per annum.

Solution 2

Given: P = Rs. 10,800; Time = $2\frac{1}{2}$ years and Rate= 10%p.a

For 2years

$$A = P\left(1+\frac{r}{100}\right)^{n} = 10,800\left(1+\frac{10}{100}\right)^{2} = Rs13,068$$

For ½ year

$$\therefore \quad A = P\left(1+\frac{r}{2\times 100}\right)^{n\times 2} = 13,068\left(1+\frac{10}{2\times 100}\right)^{\frac{1}{2}\times 2}$$

$$= 13,068\times\frac{21}{20} = 13,721.40 = Rs.13721(\text{nearest rupee})$$

$\therefore$ Rs.13,721 - Rs.10,800 = Rs.2,921

Question 3

The value of a machine, purchased two years ago, depreciates at the annual rate of 10%. If its present value is Rs.97,200, find:

i. Its value after 2 years.
ii. Its value when it was purchased.

Solution 3

(i) Present value of machine(P) = Rs.97,200
Depreciation rate = 10%

$$\therefore \text{ Value of machine after 2 years } = P\left(1-\frac{r}{100}\right)^{n}$$

$$= 97,200\left(1-\frac{10}{100}\right)^{2}$$

$$= 97,200\left(\frac{9}{10}\right)^{2}$$

=Rs.78732

(ii) Present value of machine(A) = Rs.97,200
Depreciation rate = 10% and time = 2 years
To calculate the cost 2 years ago

$$\therefore \quad A = P\left(1-\frac{r}{100}\right)^{n}$$

$$\Rightarrow 97,200 = P\left(1-\frac{10}{100}\right)^{2}$$

$$\Rightarrow 97,200 = P\left(\frac{9}{10}\right)^{2}$$

$$\Rightarrow P = Rs.\ 97,200\times\left(\frac{10}{9}\right)^{2} = 1,20,000$$

Question 4

Anuj and Rajesh each lent the same sum of money for 2 years at 8% simple interest and compound interest respectively. Rajesh received Rs.64 more than Anuj. Find the money lent by each and interest received.

Solution 4

Let the sum of money lent by both Rs.y

For Anuj

P = Rs.y ;rate = 8% and time = 2 years

$$\therefore \text{ S.I. } = \frac{P \times R \times T}{100} = \frac{y \times 8 \times 2}{100} = \frac{4y}{25}$$

For Rajesh

P = Rs.y ;rate = 8% and time = 2 years

$$\therefore \text{ C.I. } = P\left[\left(1+\frac{r}{100}\right)^n - 1\right] = y\left[\left(1+\frac{8}{100}\right)^2 - 1\right] = \frac{104y}{625}$$

Given : C.I. -Rs.64

$$\Rightarrow \frac{104y}{625} - \frac{4y}{25} = 64$$

$$\Rightarrow \frac{4y}{625} = 64 \Rightarrow y = \frac{64 \times 625}{4} = \text{Rs.}10,000$$

$$\text{Interest received by Anuj } = \frac{4 \times 10,000}{25} = \text{Rs.}1600$$

$$\text{Interest received by Rajesh} = \frac{104 \times 10,000}{625} = \text{Rs. } 1664$$

Question 5

Calculate the sum of money on which the compound interest (payable annually) for 2 years be four times the simple interest on Rs.4,715 for 5 years, both at the rate of 5% per annum.

Solution 5

Given: Principal = Rs.4,715;time = 5 years and rate= 5% p.a.

$$\therefore \text{ S.I. } = \frac{P \times R \times T}{100} = \frac{4715 \times 5 \times 5}{100} = 1,178.75$$

Then C.I. = Rs.1,178.75 x 4 = Rs.4,715

Time = 2 years and rate = 5%

$$\therefore \text{ C.I. } = P\left[\left(1+\frac{r}{100}\right)^n - 1\right]$$

$$\Rightarrow 4,715 = P\left[\left(1+\frac{5}{100}\right)^2 - 1\right]$$

$$\Rightarrow 4,715 = P\left(\frac{41}{400}\right)$$

$$\Rightarrow P = \text{Rs}\frac{4,715 \times 400}{41} = \text{Rs. } 46,000$$

Question 6

A sum of money was invested for 3 years, interest being compounded annually. The rates for successive years were 10%, 15% and 18% respectively. If the compound interest for the second year amounted to Rs.4,950, find the sum invested.

Solution 6

Given: C.I. for the 2nd year = Rs.4,950 and rate = 15%

$$\text{Then, C.I.} = P\left[\left(1+\frac{r}{100}\right)^{n}-1\right]$$

$$\Rightarrow \quad 4{,}950 = P\left[\left(1+\frac{15}{100}\right)^{1}-1\right]$$

$$\Rightarrow \quad 4{,}950 = P\left(\frac{3}{20}\right)$$

$$\Rightarrow \quad P = \frac{4{,}950 \times 20}{3}$$

$$\Rightarrow \quad P = \text{Rs. } 33{,}000$$

Then amount at the end of 2nd year= Rs.33,000

For first 2years

A = Rs.33,000; r_1 =10%

$$\therefore \quad A = P\left(1+\frac{r_1}{100}\right)$$

$$\Rightarrow \quad 33{,}000 = P\left(1+\frac{10}{100}\right)$$

$$\Rightarrow \quad 33{,}000 = P\left(\frac{11}{10}\right)$$

$$\Rightarrow \quad P = \frac{33{,}000 \times 10}{11} = 30{,}000$$

The sum invested is Rs.30,000.

Question 7

A sum of money is invested at 10% per annum compounded half yearly. If the difference of amounts at the end of 6 months and 12 months is Rs.189, find the sum of money invested.

Solution 7

Let the sum of money be Rs.y

and rate = 10% p.a. compounded half yearly

For first 6months

$$\therefore \quad A = P\left(1+\frac{r}{2\times 100}\right)^{n\times 2} = y\left(1+\frac{10}{2\times 100}\right)^{\frac{1}{2}\times 2} = \left(\frac{21}{20}\right)y$$

For first 12 months

$$\therefore \quad A = P\left(1+\frac{r}{2\times 100}\right)^{n\times 2} = y\left(1+\frac{10}{2\times 100}\right)^{1\times 2} = \left(\frac{441}{400}\right)y$$

Given: The difference between the above amounts = Rs.189

$$\Rightarrow \left(\frac{441}{400}\right)y - \left(\frac{21}{20}\right)y = 189$$

$$\Rightarrow \left(\frac{21}{400}\right)y = 189$$

$$\Rightarrow y = \frac{189 \times 400}{21}$$

y = 3600

Question 8

Rohit borrows Rs.86,000 from Arun for two years at 5% per annum simple interest. He immediately lends out this money to Akshay at 5% compound interest compounded annually for the same period. Calculate Rohit's profit in the transaction at the end of two years.

Solution 8

P = Rs.86,000;time = 2 years and rate = 5% p.a.

To calculate S.I.

$$\therefore \quad S.I. = \frac{P \times R \times T}{100} = \frac{86,000 \times 5 \times 2}{100} = Rs.\ 8,600$$

To calculate C.I.

$$\therefore \quad C.I. = P\left[\left(1 + \frac{r}{100}\right)^n - 1\right]$$

$$= 86,000\left[\left(1 + \frac{5}{100}\right)^2 - 1\right]$$

$$= 86,000\left(\frac{41}{400}\right) = Rs.\ 8,815$$

Profit = C.I. - S.I. = Rs.8,815 - Rs.8,600 = Rs.215

Question 9

The simple interest on a certain sum of money for 3 years at 5% per annum is Rs.1,200. Find the amount and the compound interest due on this sum of money at the same rate and after 2 years. Interest is reckoned annually.

Solution 9

Let Rs.x be the sum of money.

Rate = 5 % p.a. Simple interest = Rs.1,200, n = 3years.

$$1,200 = \frac{x \times 5 \times 3}{100}$$

$$\Rightarrow x = \frac{12,00,00}{15}$$

$$\Rightarrow x = 8,000$$

The amount due and the compound interest on this sum of money at the same rate and after 2 years.

P = Rs.8,000;rate = 5% p.a., n = 3 years

$$\therefore A = P\left(1+\frac{r}{100}\right)^n$$

$$\Rightarrow A = 8,000\left(1+\frac{5}{100}\right)^2$$

$$\Rightarrow A = 8,000(1.1025)$$

$$\Rightarrow A = 8,820$$

$$C.I. = A - P$$

$$\Rightarrow C.I. = 8,820 - 8,000$$

$$\Rightarrow C.I. = 820$$

The amount due after 2 years is Rs.8,820 and the compound interest is Rs.820.

Question 10

Nikita invests Rs.6,000 for two years at a certain rate of interest compounded annually. At the end of first year it amounts to Rs.6,720. Calculate:
(a) The rate of interest.
(b) The amount at the end of the second year.

Solution 10

Let x% be the rate of interest.

P = Rs.6,000, n = 2 years, A = Rs.6,720

i. For the first year

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$\Rightarrow 6,720 = 6,000\left(1+\frac{x}{100}\right)^1$$

$$\Rightarrow 6,720 - 6,000 = 60x$$

$$\Rightarrow x = 12$$

The rate of interest is x% = 12%.

ii. The amount at the end of the second year.

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$\Rightarrow A = 6,000\left(1+\frac{12}{100}\right)^2$$

$$\Rightarrow A = 6,000\left(\frac{112}{100}\right)^2$$

$$\Rightarrow A = 7,526.40$$

The amount at the end of the second year = Rs.7,526.40